Chapter 9. Supply Chain Models

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9.1. Introduction

Model Characteristics

Multicommodity

Multi-echelon

Capacitated Facilities

Capacitated Channels

Deterministic or Stochastic

Single and Multiple Periods

Model and Algorithm Hierarchy

Evaluate or Benchmark

Digital simulation or spreadsheet or maps

Distribution Channel Selection

Current facilities

Select best distribution channel and set inventory levels

Spreadsheet or custom programming

Network Optimization

Current facilities and capacities

Optimize the product flow

Linear Programming solver

Location-Allocation

Moving Facilities

Current facility status

No site-dependent costs

Cost proportional to material flow quantity, such as transportation and material handling in the distribution center

Alternative generating algorithm

Limited location resolution (at the level of a county or metropolitan area)

Approximate algorithms

Non-linear optimization, specialized heuristics

Production-Distribution

Everything is allowed to change

Alternative selecting

Site dependent costs

Mixed Integer Programming solver

Production-Distribution Models and Algorithms

Kuehn and Hamburger
Drop - Add- Swap heuristic

Geoffrion and Graves

Benders' decomposition

Goetschalckx and Wei

Disaggregated MIP Formulation with Branch and Bound with LP relaxation

9.2. Transportation Mode Selection Model

Parameters and Variables

HCF	inventory holding cost factor (dollars per dollar of inventory per year)
D_p	annual demand for product p
v_p	value of a unit of product p
pc_p	unit production cost of product p
TT_m	transit time of transportation mode m (expressed in years)
tc_{mp}	transportation cost for shipping one unit of product p with transportation mode m
TB_{mp}	transportation batch size of product p shipped with transportation mode m
sc_p	annualized fixed storage cost per unit of product p
PC_p	total annual production cost for product p
TC_{mp}	total annual transportation cost for product p shipped with transportation mode m
pic_{mp}	unit pipeline inventory cost of product p shipped with transportation mode m
PIC_{mp}	pipeline inventory cost of product p shipped with transportation mode m
$OCIC_{imp}$	cycle inventory cost of product p shipped with transportation mode m from plant i
$DCIC_{jmp}$	cycle inventory cost of product p shipped with transportation mode m to distribution center j
d_p	average demand during observable demand period, e.g. daily demand, of product p
Vd_p	variance of the demand during observable demand period, e.g. daily demand, of product p
CVd_p	coefficient of variation of the demand during observable demand period, e.g. daily demand, of product p

LT_{mp}	average lead time for the delivery of product *p* using transportation mode *m* expressed in observable demand periods, e.g. days
VLT_{mp}	variance of the lead time for the delivery of product *p* using transportation mode *m* expressed in observable demand periods, e.g. days
SI_{mp}	safety inventory of product *p* shipped with transportation mode *m*
$SSIC_{jmp}$	safety stock inventory cost in distribution center *j* of product *p* shipped with transportation mode *m*
FSC_{jmp}	total annual fixed storage cost in distribution center *j* for product *p* shipped with transportation mode *m*
TIC_{p}	total invariant cost for product *p*
TVC_{mp}	total variable cost for product *p* using transportation mode *m*
TFC_{mp}	total fixed cost for product *p* using transportation mode *m*
TAC_{mp}	total aggregate cost for product *p* using transportation mode *m*

A typical value of the holding cost factor or HCF is 0.25, i.e., 25 cents per dollar of inventory per year. To find the holding cost factor for another time period, we have to divide by the number of time periods in a year. A HCF equal to 0.25 per year equals a holding cost factor of 0.25/365 = 0.000685 per day.

Demand During Lead Time

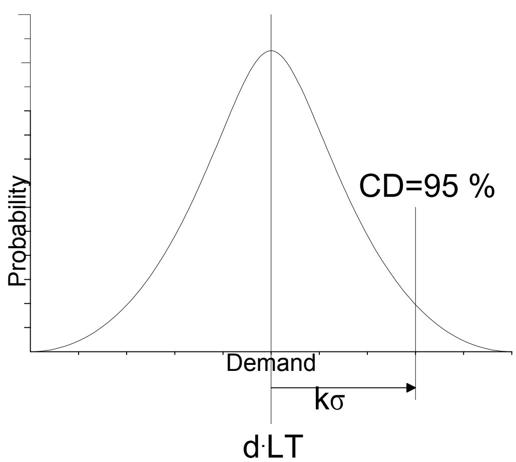


Figure 9.1. Demand During Lead Time Distribution

Assuming single sourcing and a single transportation mode of the supply of each product at the distribution centers, the safety inventory at the distribution center can then be computed as

$$SI = k \cdot \sqrt{LT \cdot Var_d + d^2 \cdot Var_{LT}} \tag{9.1}$$

$$CV_d = \frac{\sqrt{Var_d}}{d}$$
$$Var_d = \left(CV_d \cdot d\right)^2 \tag{9.2}$$

$$SI = k \cdot \sqrt{LT \cdot CV_d^2 + Var_{LT}} \cdot d \tag{9.3}$$

This formula quantifies the common practice in industry of keeping a safety stock level on hand equal to a number of demand periods. The formula shows the relationship between the safety inventory and the customer service level based on probability of delivery out on-hand inventory, the average and variance of the lead time, and the average and variance of the demand. Since the demand is determined by the customers and the service level is usually a mandate from corporate management, the only factors that can be influenced by the warehouse manager are the average and variance of the lead time and the variance of the demand. Safety inventory can be reduced if the input and output flows, i.e., supply and demand, are kept as constant as possible and if the lead time for replenishments is reduced.

Cost Computations

Invariant Costs

Invariant costs are costs that are incurred by the logistics systems but that do not depend on the selection of the distribution channel. Since, it is assumed that the total demand does not depend on the distribution channel, the total production cost is a member of the invariant costs. The total production costs is computed as product of the yearly demand and the unit production costs.

$$PC_p = D_p \cdot pc_p \tag{9.4}$$

$$TIC_p = PC_p \tag{9.5}$$

Fixed Costs

Fixed costs are costs whose magnitude does not change during the operation of the distribution channel. The fixed costs may be different from one distribution channel to another, but once a channel selection has been made, the fixed costs remain unchanged. In distribution channel design, the size of the warehouse remains unchanged for extended periods of time. Hence, the annualized cost per warehouse location or per cubic foot of warehouse space is a component of the fixed costs. The space occupied by a product is typically computed as proportional to the maximum inventory of this product. The maximum inventory for each product is the sum of the cycle and safety inventory. The total fixed storage cost is then the product of the maximum product inventory and the annualized unit storage cost for that product.

$$FSC_{jmp} = \left[TB_{mp} + SI_{mp} \right] \cdot sc_p \tag{9.6}$$

$$TFC_{mp} = FSC_{jmp} \tag{9.7}$$

Variable Costs

The variable costs are costs that may change during the operation of the distribution channel. Typically these costs are a function of either the transfer batch size or order quantity and of the total annual demand.

The total transportation cost is computed as the product of the annual demand and the unit transportation cost.

$$TC_{mp} = D_p \cdot tc_{mp} \tag{9.8}$$

The pipeline inventory cost is computed as the product of the annual demand, the value of single unit at the origin of the pipeline, the transit time for the flow to go through the pipeline, and the inventory holding cost rate.

$$pic_{mp} = v_p \cdot TT_m \cdot HCF \tag{9.9}$$

$$PIC_{mp} = D_p \cdot v_p \cdot TT_m \cdot HCF \tag{9.10}$$

When shipping in transportation batches, there is both an inventory build up at the source until the shipment occurs, and inventory depletion at the destination starting after a replenishment shipment arrives. If we assume a constant build up and depletion rate, then the average inventory follows the classical saw tooth pattern and the average inventory is half the maximum inventory or transportation batch size. The value of one unit of product is different at the origin and destination. At the destination the corporation has invested the sum of the production cost, the transportation cost, and the in-transit inventory cost in the product. The origin and destination cycle inventory costs are then computed as the product of half the transportation batch size, the holding cost rate, and the appropriate unit value.

$$OCIC_{imp} = \left(TB_{mp}/2\right) \cdot v_p \cdot HCF \tag{9.11}$$

$$DCIC_{imp} = \left(TB_{mp}/2\right) \cdot \left(v_p + tc_{mp} + pic_{mp}\right) \cdot HCF \tag{9.12}$$

The safety inventory level is computed with the assumption of a linear safety inventory policy as expressed in (9.3). The safety inventory cost is then computed as the product of the safety inventory level, the value for each unit, and the holding cost rate.

$$SI_{mp} = k\sqrt{LT_{mp}CVd_p^2 + VLT_{mp}}d_p \tag{9.13}$$

$$SIC_{mp} = SI_{mp} \cdot \left(v_p + tc_{mp} + pic_{mp}\right) \cdot HCF \tag{9.14}$$

The total variable cost is computed as the sum of the transportation cost, the pipeline inventory cost, the origin and destination cycle inventory costs, and the safety inventory cost.

$$TVC_{mp} = TC_{mp} + PIC_{mp} + OCIC_{imp} + DCIC_{jmp} + SIC_{jmp} \tag{9.15}$$

Finally, the total aggregate cost is computed as the sum of the total invariant, variable, and fixed costs.

$$TAC_{mp} = TIC_p + TVC_{mp} + TFC_{mp} \tag{9.16}$$

Ballou Channel Selection Example

The calculations for the channel selection example in Ballou (1998) are summarized in the next table. The safety inventory levels in the plant and the distribution center are not computed with the standard formulas shown above, put are based on description given in Ballou. For example, the average inventory in the distribution center when shipping by piggyback is specified as 0.93 * 50,000 = 46,500. Since the average cycle inventory is 35,000 / 2 = 17,5000, the average safety inventory is then 46,500 – 17,500 = 29,000. The safety inventory levels for the other transportation modes were calculated in a similar way.

There were no warehouse storage costs included in the example. Based on the given parameters, truck transportation is the preferred distribution transportation mode.

Table 9.1. Channel Selection Calculations for the Ballou Example

Annual Demand	700000
Annual Holding Cost Rate	0.3
Unit Production Cost	$30.00
Unit Annualized Warehouse Cost	$0.00

	Rail	Piggyback	Truck	Air
Unit Transportation Cost ($)	0.1	0.15	0.2	1.4
Channel Transit Time (days)	21	14	5	2
Transportation Batch Size	70000	35000	35000	17500

	Rail	Piggyback	Truck	Air
Production Cost	$21,000,000	$21,000,000	$21,000,000	$21,000,000
Total Invariant Costs	**$21,000,000**	**$21,000,000**	**$21,000,000**	**$21,000,000**
Transportation Costs	$70,000	$105,000	$140,000	$980,000
In-Transit Inventory	$362,466	$241,644	$86,301	$34,521
Order Fequency per Year	10	20	20	40
Order Cycle Length in Years	0.1	0.05	0.05	0.025
Plant Max Cycle Inventory	70,000	35,000	35,000	17,500
Plant Cycle Inventory Costs	$315,000	$157,500	$157,500	$78,750
Plant Safety Inventory	65,000	29,000	24,500	11,250
Plant Safety Inventory Costs	$585,000	$261,000	$220,500	$101,250
Unit Value at DC	$30.62	$30.50	$30.32	$31.45
DC Max Cycle Inventory	70,000	35,000	35,000	17,500
DC Cycle Inventory Costs	$321,487	$160,100	$159,197	$82,554
DC Safety Inventory	65,000	29,000	24,500	11,250
DC Safety Inventory Costs	$597,047	$265,308	$222,876	$106,141
DC Maximum Inventory	135,000	64,000	59,500	28,750
Total Marginal Costs	**$2,251,000**	**$1,190,552**	**$986,375**	**$1,383,216**
Annualized Warehouse Costs	$0	$0	$0	$0
Total Variant Costs	**$2,251,000**	**$1,190,552**	**$986,375**	**$1,383,216**
Total Cost	**$23,251,000**	**$22,190,552**	**$21,986,375**	**$22,383,216**

9.3. Kuehn and Hamburger Model

Model Characteristics

Multicommodity

Zero echelon

Uncapacitated depots

Depot Handling Cost possible as an extension

Deterministic

Single Period

Arc or path formulation are equivalent since this is a zero echelon model.

“Weak” formulation with aggregate consistency or linkage constraints.

Model

$$\begin{aligned} \min \quad & z = \sum_{j=1}^{N} \left(f_j y_j + \sum_{i=1}^{M} c_{ij} x_{ij} \right) \\ s.t. \quad & \sum_{j=1}^{N} x_{ij} = 1 \qquad \forall i \\ & \sum_{i=1}^{M} x_{ij} - M y_j \le 0 \qquad \forall j \\ & y_j \in \{0,1\}, x_{ij} \ge 0 \end{aligned} \tag{9.17}$$

Shadow Prices and Site Relative Costs

u_i Current cost for servicing customer *i*

Error! Objects cannot be created from editing field codes. Site relative cost for opening warehouse j based on the current customer service cost u_i. Note that both u_i and c_{ij} are the cost for servicing the total demand of a customer for a particular product.

$$\rho_j(\mathbf{U}) = f_j + \sum_{i=1}^{M} \min\left\{0, c_{ij} - u_i\right\} \tag{9.18}$$

Drop-Add-Swap Heuristic

Drop

Start with all facilities open

Close the one with the largest positive relative site cost

Until no further decrease in costs

Add

Start with all facilities closed

Open the one with the most negative relative site cost

Until no further decrease in costs

Swap

Start with drop

Swap (Open and Close) the two facilities with the most extreme relative site cost

Until not further cost reduction

Drop-Add-Swap Heuristic Example

The drop-add-swap heuristic is applied to the supply chain configuration example in Ballou (1998), pp. 498. This heuristic only can handle zero echelon supply chains, where goods flow directly from the source to the sink facilities, so the manufacturing plants are ignored in the following computations. This heuristic can only handle uncapacitated source facilities, so the capacity of warehouse *W1* is ignored. The fixed costs are $100,000 and $500,000 for warehouse 1 and 2, respectively. The cost to serve a customer is the sum of the transportation cost between the respective origin and destination facility and the handling cost in the warehouse. Let u_i and c_{ij} be the current unit cost to serve customer *i* and the marginal unit cost to serve customer *i* from warehouse *j*, respectively. Let U_i and C_{ij} be the current total cost to serve customer *i* and the total marginal cost to serve customer *i* from warehouse *j*, respectively. Note that this is a change in notation from the original notation used to define the site relative cost factor. The marginal costs can then be summarized in the Table 2.

The drop phase starts with the current customer service costs based on the actual situation or the best current cost for serving each customer, based on the solution of a standard network flow problem for each of the products with as sources all warehouses and as sinks all customer-product combinations. However, since the source facilities are uncapacitated, we can easily find the solution to the network flow problem since each customer will always be served from the cheapest open source.

Table 9.2. Drop Phase Marginal Costs for Servicing Customers

	Destination	C1P1	C2P1	C3P1	C1P2	C2P2	C2P2
	Demand	50,000.00	100,000.00	50,000.00	20,000.00	30,000.00	60,000.00
Origin							
W_1	c_{i1}	4+2=6	3+2=5	5+2=7	3+2=5	2+2=4	4+2=6
	C_{i1}	300,000.00	500,000.00	350,000.00	100,000.00	120,000.00	260,000.00
W_2	c_{i2}	2+1=3	1+1=2	2+1=3	3+1=4	2+1=3	3+1=4
	C_{i2}	150,000.00	200,000.00	150,000.00	80,000.00	90,000.00	240,000.00
	U_i	150,000.00	200,000.00	150,000.00	80,000.00	90,000.00	240,000.00

Since at the beginning of the drop phase both distribution centers are assumed to be open, the best current cost for servicing a customer is the minimum of the service cost from either warehouse. For this particular example at this particular iteration, the least expensive service is to deliver to all customers from warehouse *W2*. The total cost for the configuration at this time is then the sum of the fixed costs for all warehouse plus the sum of the current best service costs, or

$$\begin{aligned} TC &= (100000+500000)+(150000+200000+150000+80000+90000+240000) \\ &= 600000+910000=1510000 \end{aligned}$$

Note that the fixed cost of warehouse *W1* is still included in the total cost even though it is not used.

The next step is to compute the site relative cost factors for each warehouse site based on the current values of the customer service costs. With the notation adopted in this example, the site relative cost factors are computed with the following formula

$$\rho_j(\mathbf{U}) = f_j + \sum_{i=1}^{M} \min\{0, C_{ij} - U_i\}$$

$$\begin{aligned} \rho_1 &= 100000 + \begin{pmatrix} \min\{0,300000-150000\}+\min\{0,500000-200000\}+ \\ \min\{0,350000-150000\}+\min\{0,100000-80000\}+ \\ \min\{0,120000-90000\}+\min\{0,360000-240000\} \end{pmatrix} \\ &= 100000+0=100000 \end{aligned}$$

$$\begin{aligned} \rho_2 &= 500000 + \begin{pmatrix} \min\{0,150000-150000\}+\min\{0,200000-200000\}+ \\ \min\{0,150000-150000\}+\min\{0,80000-80000\}+ \\ \min\{0,90000-90000\}+\min\{0,240000-240000\} \end{pmatrix} \\ &= 500000+0=500000 \end{aligned}$$

The drop heuristic will close the facility with the largest positive site relative cost, which in this case is warehouse *W2* with a site relative cost of 500,000. The total cost for this new configuration is $1,830,000, as computed by the next formula.

$$TC = (100000) + (300000 + 500000 + 350000 + 100000 + 120000 + 360000)$$
$$= 100000 + 1730000 = 1830000$$

This is more expensive than the previous configuration of the drop phase and so the drop phase would not close warehouse *W2* and the drop phase terminates.

If closing warehouse *W2* would have reduced the total cost, then the customer service costs have to be recomputed based on the current configuration. Since the only warehouse facility open at this time would be *W1*, the updated customer service costs would be the marginal service costs from facility *W1*. At this point only the single warehouse *W1* would remain open and the drop heuristic would terminate. In larger supply chain systems the drop phase may go through several iterations, closing during each iteration the facility with the largest positive site relative cost factor and then recomputing the customer service costs.

The add phase starts with the current customer service costs based on the actual situation or with a single open distribution center. From earlier calculations, the best warehouse to open based on the marginal customer service costs is depot *W2*. The customer service costs are shown in Table 2. The site relative costs based on these customer service costs have already been computed and none of the site relative costs are negative, so the add heuristic does not add a facility and terminates. The total cost at the end of the add phase is $1,410,000.

$$TC = (500000) + (150000 + 200000 + 150000 + 80000 + 90000 + 240000)$$
$$= 500000 + 910000 = 1410000$$

The swap phase attempts to improve the total cost by executing first the drop phase and then closing one facility and simultaneously opening one additional facility. The facility selected for closure is the open facility with the largest positive relative site cost factor. The facility selected for opening is the closed facility with the smallest relative site cost factor. For this particular example, this would entail closing warehouse *W2* and not opening an additional warehouse since all warehouses are still open. This configuration has already been examined and the total cost would be $1,830,000 and the swap phase terminates without making a single exchange.

The overall heuristic terminates with the best configuration after the swap phase, which in this particular example consists of warehouse *W2* open and servicing all customers with all products and warehouse *W1* is closed for a total cost of $1,410,000.

If the add phase would have been started with the single warehouse *W1* open, then the total cost would have been $1,830,000 and the site relative cost for warehouse *W2* would have been (negative) -320,000 and warehouse *W2* would have been added. The new configuration would have a total cost of $1,510,000 and would have all warehouses open, which is equivalent tot the starting configuration of the drop phase. The swap phase would not be able to make any exchanges and the heuristic terminates with both warehouses open, which is clearly not optimal. Obviously, this example is too small to demonstrate the normal iterations of the Drop-Add-Swap heuristic.

9.4. Arc-Based Model Solved with Mixed Integer Programming

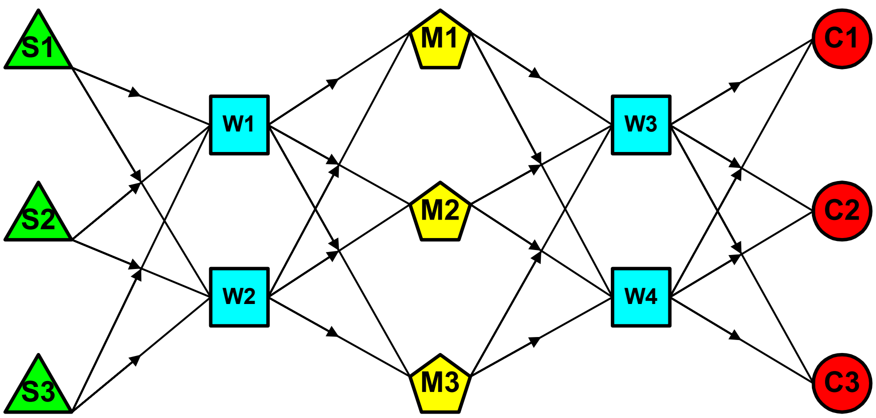


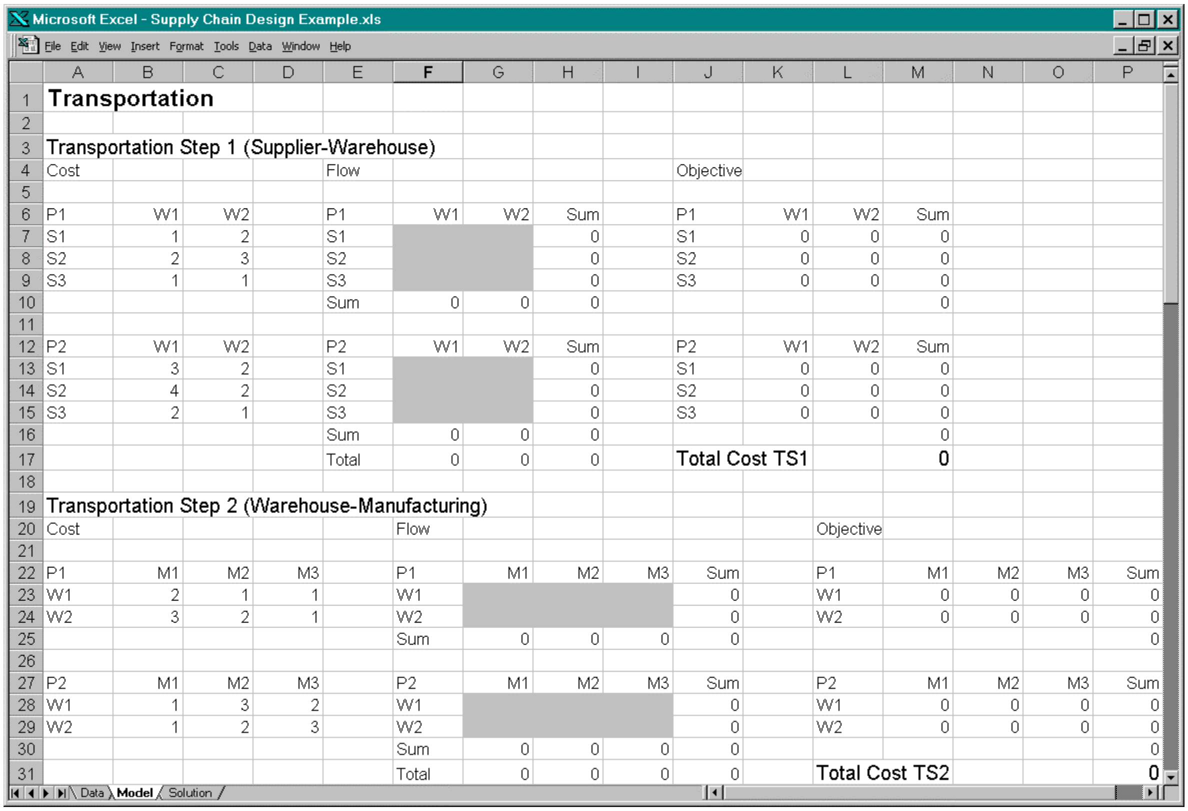
Figure 9.2. Arc-Based Multi-echelon Supply Chain Example Illustration

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Cost Data**											
2												
3	Transportation Step 1 (Supplier-Warehouse)						Transportation Step 3 (Manufacturing-Warehousing)					
4												
5	P1	W1	W2				P1	W3	W4			
6	S1	1	2				M1	2	1			
7	S2	2	3				M2	3	2			
8	S3	1	1				M3	2	3			
9												
10	P2	W1	W2				P2	W3	W4			
11	S1	3	2				M1	1	2			
12	S2	4	2				M2	2	1			
13	S3	2	1				M3	3	4			
14												
15	Transportation Step 2 (Warehouse-Manufacturing)						Transportation Step 4 (Warehouse-Customer)					
16												
17	P1	M1	M2	M3			P1	C1	C2	C3		
18	W1	2	1	1			W3	2	3	2		
19	W2	3	2	1			W4	1	2	3		
20												
21	P2	M1	M2	M3			P2	C1	C2	C3		
22	W1	1	3	2			W3	1	2	3		
23	W2	1	2	3			W4	2	1	4		

Figure 9.3. Arc-Based Supply Chain Example Transportation Data

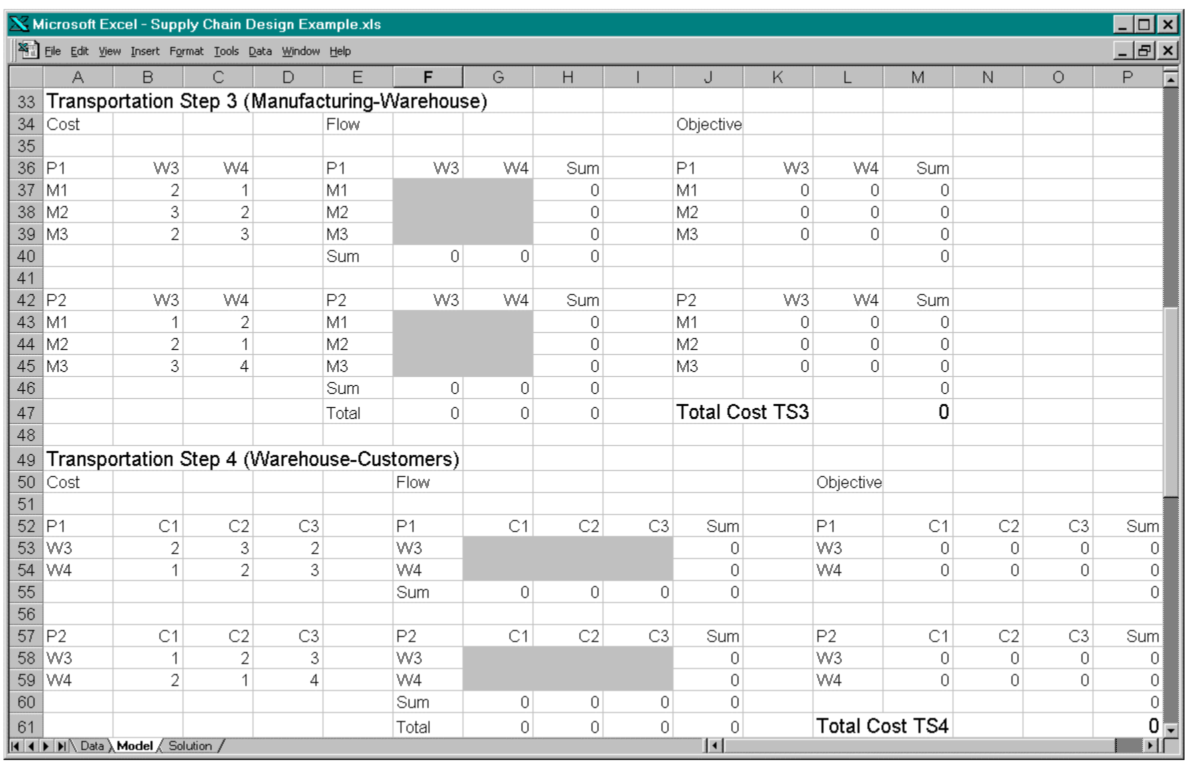
	A	B	C	D	E	F	G	H	I
47	**Capacities**					**Customer Demands**			
48									
49	Supplier Capacity						C1	C2	C3
50						P1	100	75	25
51	S1	300				P2	100	100	40
52	S2	400							
53	S3	200							
54									
55	Warehousing Capacity (Throughput)					**Facility Costs**			
56									
57	W1	300				S1	100		
58	W2	350				S2	200		
59	W3	300				S3	300		
60	W4	350				W1	50		
61						W2	80		
62	Manufacturing					M1	250		
63						M2	200		
64	M1	500				M3	50		
65	M2	400				W3	50		
66	M3	300				W4	80		

Figure 9.4. Arc-Based Supply Chain Example Facility Capacity and Cost Data



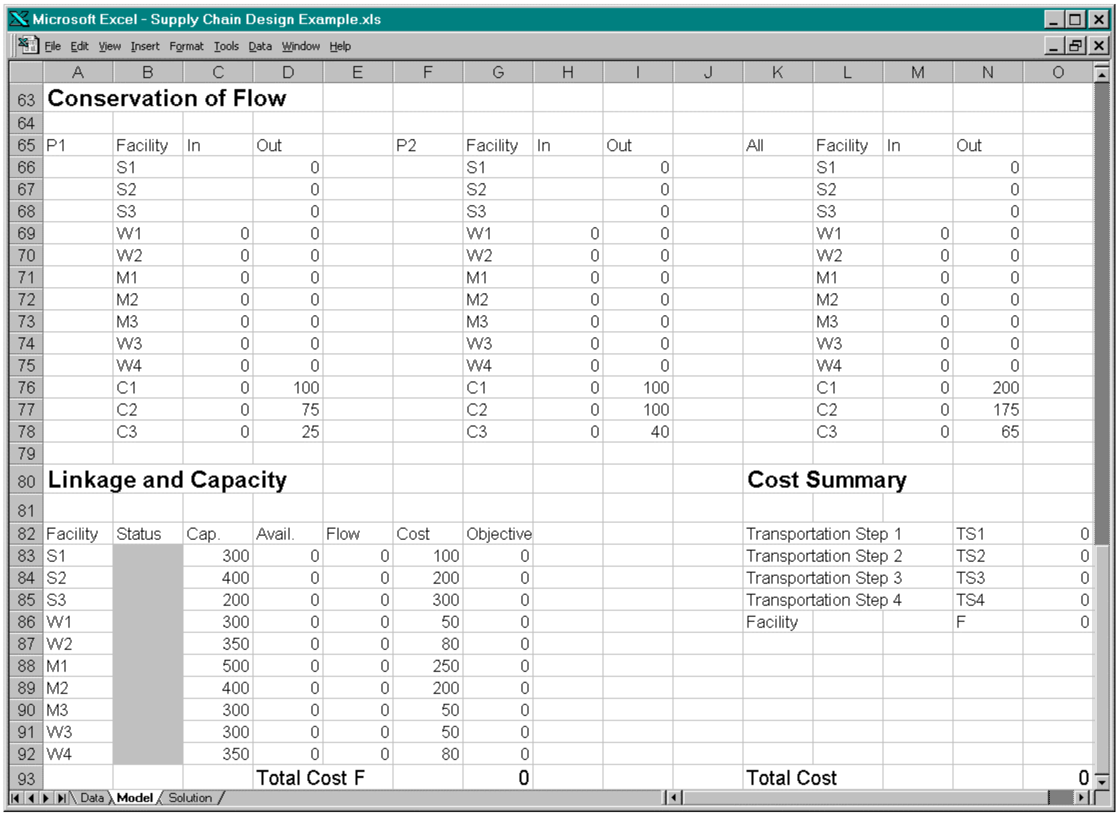
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Transportation															
2																
3	Transportation Step 1 (Supplier-Warehouse)															
4	Cost				Flow					Objective						
5																
6	P1	W1	W2		P1	W1	W2	Sum		P1	W1	W2	Sum			
7	S1	1	2		S1			0		S1	0	0	0			
8	S2	2	3		S2			0		S2	0	0	0			
9	S3	1	1		S3			0		S3	0	0	0			
10					Sum	0	0	0					0			
11																
12	P2	W1	W2		P2	W1	W2	Sum		P2	W1	W2	Sum			
13	S1	3	2		S1			0		S1	0	0	0			
14	S2	4	2		S2			0		S2	0	0	0			
15	S3	2	1		S3			0		S3	0	0	0			
16					Sum	0	0	0					0			
17					Total	0	0	0		Total Cost TS1			0			
18																
19	Transportation Step 2 (Warehouse-Manufacturing)															
20	Cost					Flow						Objective				
21																
22	P1	M1	M2	M3		P1	M1	M2	M3	Sum		P1	M1	M2	M3	Sum
23	W1	2	1	1		W1				0		W1	0	0	0	0
24	W2	3	2	1		W2				0		W2	0	0	0	0
25						Sum	0	0	0	0						0
26																
27	P2	M1	M2	M3		P2	M1	M2	M3	Sum		P2	M1	M2	M3	Sum
28	W1	1	3	2		W1				0		W1	0	0	0	0
29	W2	1	2	3		W2				0		W2	0	0	0	0
30						Sum	0	0	0	0						0
31						Total	0	0	0	0		Total Cost TS2				0

Figure 9.5. Arc-Based Supply Chain Example Transportation Supplier-Manufacturing Model



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
33	Transportation Step 3 (Manufacturing-Warehouse)															
34	Cost				Flow					Objective						
35																
36	P1	W3	W4		P1	W3	W4	Sum		P1	W3	W4	Sum			
37	M1	2	1		M1			0		M1	0	0	0			
38	M2	3	2		M2			0		M2	0	0	0			
39	M3	2	3		M3			0		M3	0	0	0			
40					Sum	0	0	0					0			
41																
42	P2	W3	W4		P2	W3	W4	Sum		P2	W3	W4	Sum			
43	M1	1	2		M1			0		M1	0	0	0			
44	M2	2	1		M2			0		M2	0	0	0			
45	M3	3	4		M3			0		M3	0	0	0			
46					Sum	0	0	0					0			
47					Total	0	0	0		Total Cost TS3			0			
48																
49	Transportation Step 4 (Warehouse-Customers)															
50	Cost					Flow						Objective				
51																
52	P1	C1	C2	C3		P1	C1	C2	C3	Sum		P1	C1	C2	C3	Sum
53	W3	2	3	2		W3				0		W3	0	0	0	0
54	W4	1	2	3		W4				0		W4	0	0	0	0
55						Sum	0	0	0	0						0
56																
57	P2	C1	C2	C3		P2	C1	C2	C3	Sum		P2	C1	C2	C3	Sum
58	W3	1	2	3		W3				0		W3	0	0	0	0
59	W4	2	1	4		W4				0		W4	0	0	0	0
60						Sum	0	0	0	0						0
61						Total	0	0	0	0		Total Cost TS4				0

Figure 9.6. Arc-Based Supply Chain Example Transportation Manufacturing-Customer Model



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
63	Conservation of Flow														
64															
65	P1	Facility	In	Out		P2	Facility	In	Out		All	Facility	In	Out	
66		S1		0			S1		0			S1		0	
67		S2		0			S2		0			S2		0	
68		S3		0			S3		0			S3		0	
69		W1	0	0			W1	0	0			W1	0	0	
70		W2	0	0			W2	0	0			W2	0	0	
71		M1	0	0			M1	0	0			M1	0	0	
72		M2	0	0			M2	0	0			M2	0	0	
73		M3	0	0			M3	0	0			M3	0	0	
74		W3	0	0			W3	0	0			W3	0	0	
75		W4	0	0			W4	0	0			W4	0	0	
76		C1	0	100			C1	0	100			C1	0	200	
77		C2	0	75			C2	0	100			C2	0	175	
78		C3	0	25			C3	0	40			C3	0	65	
79															
80	Linkage and Capacity										Cost Summary				
81															
82	Facility	Status	Cap.	Avail.	Flow	Cost	Objective				Transportation Step 1			TS1	0
83	S1		300	0	0	100	0				Transportation Step 2			TS2	0
84	S2		400	0	0	200	0				Transportation Step 3			TS3	0
85	S3		200	0	0	300	0				Transportation Step 4			TS4	0
86	W1		300	0	0	50	0				Facility			F	0
87	W2		350	0	0	80	0								
88	M1		500	0	0	250	0								
89	M2		400	0	0	200	0								
90	M3		300	0	0	50	0								
91	W3		300	0	0	50	0								
92	W4		350	0	0	80	0								
93				Total Cost F			0				Total Cost				0

Figure 9.7. Arc-Based Supply Chain Example Facility Capacity Model

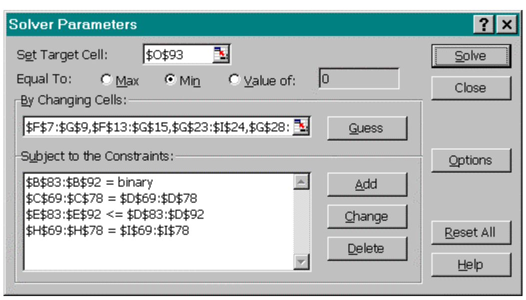


Figure 9.8. Arc-Based Supply Chain Example Solver

Microsoft Excel - Supply Chain Design Example.xls

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Transportation**															
2																
3	Transportation Step 1 (Supplier-Warehouse)															
4	Cost				Flow					Objective						
5																
6	P1	W1	W2		P1	W1	W2	Sum		P1	W1	W2	Sum			
7	S1	1	2		S1	200	0	200		S1	200	0	200			
8	S2	2	3		S2	0	0	0		S2	0	0	0			
9	S3	1	1		S3	0	0	0		S3	0	0	0			
10					Sum	200	0	200					200			
11																
12	P2	W1	W2		P2	W1	W2	Sum		P2	W1	W2	Sum			
13	S1	3	2		S1	0	40	40		S1	0	80	80			
14	S2	4	2		S2	0	0	0		S2	0	0	0			
15	S3	2	1		S3	0	200	200		S3	0	200	200			
16					Sum	0	240	240					280			
17					Total	200	240	440		Total Cost TS1			480			
18																
19	Transportation Step 2 (Warehouse-Manufacturing)															
20	Cost					Flow						Objective				
21																
22	P1	M1	M2	M3		P1	M1	M2	M3	Sum		P1	M1	M2	M3	Sum
23	W1	2	1	1		W1	200	0	0	200		W1	400	0	0	400
24	W2	3	2	1		W2	0	0	0	0		W2	0	0	0	0
25						Sum	200	0	0	200						400
26																
27	P2	M1	M2	M3		P2	M1	M2	M3	Sum		P2	M1	M2	M3	Sum
28	W1	1	3	2		W1	0	0	0	0		W1	0	0	0	0
29	W2	1	2	3		W2	240	0	0	240		W2	240	0	0	240
30						Sum	240	0	0	240						240
31						Total	440	0	0	440		Total Cost TS2				640

Data / Model / Solution

Figure 9.9. Arc-Based Supply Chain Example Transportation Supplier-Manufacturing Solution

Microsoft Excel - Supply Chain Design Example.xls

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
33	Transportation Step 3 (Manufacturing-Warehouse)															
34	Cost				Flow					Objective						
35																
36	P1	W3	W4		P1	W3	W4	Sum		P1	W3	W4	Sum			
37	M1	2	1		M1	25	175	200		M1	50	175	225			
38	M2	3	2		M2	0	0	0		M2	0	0	0			
39	M3	2	3		M3	0	0	0		M3	0	0	0			
40					Sum	25	175	200					225			
41																
42	P2	W3	W4		P2	W3	W4	Sum		P2	W3	W4	Sum			
43	M1	1	2		M1	140	100	240		M1	140	200	340			
44	M2	2	1		M2	0	0	0		M2	0	0	0			
45	M3	3	4		M3	0	0	0		M3	0	0	0			
46					Sum	140	100	240					340			
47					Total	165	275	440		Total Cost TS3			565			
48																
49	Transportation Step 4 (Warehouse-Customers)															
50	Cost					Flow						Objective				
51																
52	P1	C1	C2	C3		P1	C1	C2	C3	Sum		P1	C1	C2	C3	Sum
53	W3	2	3	2		W3	0	0	25	25		W3	0	0	50	50
54	W4	1	2	3		W4	100	75	0	175		W4	100	150	0	250
55						Sum	100	75	25	200						300
56																
57	P2	C1	C2	C3		P2	C1	C2	C3	Sum		P2	C1	C2	C3	Sum
58	W3	1	2	3		W3	100	0	40	140		W3	100	0	120	220
59	W4	2	1	4		W4	0	100	0	100		W4	0	100	0	100
60						Sum	100	100	40	240						320
61						Total	200	175	65	440		Total Cost TS4				620

Data / Model / Solution

Figure 9.10. Arc-Based Supply Chain Example Transportation Manufacturing-Customer Solution

Microsoft Excel - Supply Chain Design Example.xls

Conservation of Flow

P1 Facility	In	Out	P2 Facility	In	Out	All Facility	In	Out
S1		200	S1		40	S1		240
S2		0	S2		0	S2		0
S3		0	S3		200	S3		200
W1	200	200	W1	0	0	W1	200	200
W2	0	0	W2	240	240	W2	240	240
M1	200	200	M1	240	240	M1	440	440
M2	0	0	M2	0	0	M2	0	0
M3	0	0	M3	0	0	M3	0	0
W3	25	25	W3	140	140	W3	165	165
W4	175	175	W4	100	100	W4	275	275
C1	100	100	C1	100	100	C1	200	200
C2	75	75	C2	100	100	C2	175	175
C3	25	25	C3	40	40	C3	65	65

Linkage and Capacity

Facility	Status	Cap.	Avail.	Flow	Cost	Objective
S1	1	300	300	240	100	100
S2	0	400	0	0	200	0
S3	1	200	200	200	300	300
W1	1	300	300	200	50	50
W2	1	350	350	240	80	80
M1	1	500	500	440	250	250
M2	0	400	0	0	200	0
M3	0	300	0	0	50	0
W3	1	300	300	165	50	50
W4	1	350	350	275	80	80
			Total Cost F			910

Cost Summary

Transportation Step 1	TS1	480
Transportation Step 2	TS2	640
Transportation Step 3	TS3	565
Transportation Step 4	TS4	620
Facility	F	910
Total Cost		3215

Figure 9.11. Arc-Based Supply Chain Example Facility Capacity Solution

9.5. Geoffrion and Graves Distribution Model

Model Characteristics

Multicommodity

Single echelon

Capacitated Depots (Lower and Upper Bound)

Depot Handling Cost

Deterministic

Single Period

Path Formulation

Depot Single Sourcing

"Weak" Formulation

Additional Linear Constraints in z and y

Model

$$
\begin{aligned}
\min \quad & \sum_{ijkp} c_{ijkp} x_{ijkp} + \sum_{j} \left(f_j z_j + h_j \sum_{kp} r_{kp} y_{jk} \right) & & \\
s.t. \quad & \sum_{jk} x_{ijkp} \le s_{ip} & & \forall ip \\
& \sum_{i} x_{ijkp} = r_{kp} y_{jk} & & \forall jk \\
& \sum_{j} y_{jk} = 1 & & \forall k \\
& TL_j z_j \le \sum_{pk} r_{kp} y_{jk} \le TU_j z_j & & \forall j
\end{aligned}
\tag{9.19}
$$

Benders (Primal) Decomposition

Fix binary variables z and y

Solve independent commodity network flow problems

Determine total transportation cost

Add total transportation cost as a cut to binary master problem

Solve binary master problem

Iterate

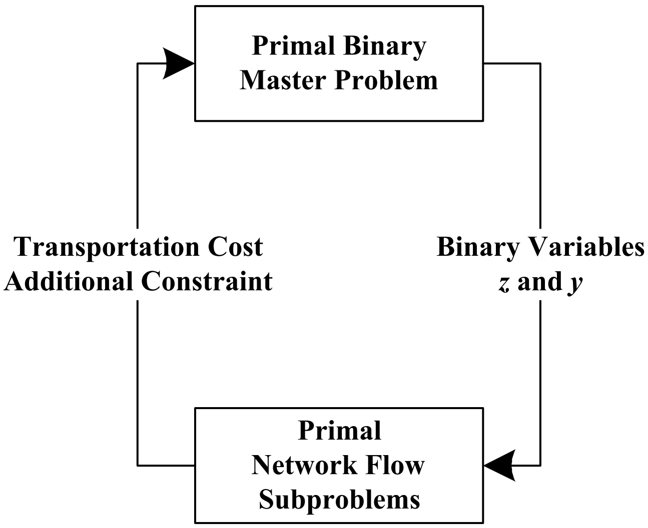


Figure 12. Benders Decomposition Flowchart

Primal Network Flow Subproblem

$$\begin{aligned}
\min \quad & \sum_{ijkp} c_{ijkp} x_{ijkp} & & \\
s.t. \quad & \sum_{jk} x_{ijkp} \le s_{ip} & & [v_{ip}] \\
& \sum_{i} x_{ijkp} = r_{kp} y_{jk} & & [u_{jkp}] \\
& x_{ijkp} \ge 0 & &
\end{aligned} \tag{9.20}$$

Note that the y are parameters in this formulation, not variables.

Dual Network Flow Subproblem

$$\begin{aligned}
\max \quad & y_0 = \sum_{ip} -v_{ip} s_{ip} + \sum_{jkp} u_{jkp} r_{kp} y_{jk} & & \\
s.t. \quad & u_{jkp} - v_{ip} \le c_{ijkp} & & \forall ijkp \\
& v_{ip} \ge 0, \;\; u_{jkp} \;\; \textit{unrestricted} & &
\end{aligned} \tag{9.21}$$

Note that the y are parameters in this formulation, not variables.

This formulation can also be written in function of the extreme points of the constraint polyhedral.

$$\max_{\left[v_{ip}^e, u_{jkp}^e\right] \in E} y_0 = \sum_{ip} -v_{ip}^e s_{ip} + \sum_{jkp} u_{jkp}^e r_{kp} y_{jk} \tag{9.22}$$

This yields the following constraints generated by the network flow subproblem for the primal integer master problem.

$$y_0 \geq \sum_{ip} -v_{ip}^e s_{ip} + \sum_{jkp} u_{jkp}^e r_{kp} y_{jk} \tag{9.23}$$

Primal Binary Master Problem

$$\begin{aligned}
\min \quad & \sum_j \left(f_j z_j + h_j \sum_{kp} r_{kp} y_{jk} \right) + y_0 \\
s.t. \quad & \sum_j y_{jk} = 1 & \forall k \\
& TL_j z_j \leq \sum_{pk} r_{kp} y_{jk} \leq TU_j z_j & \forall j \\
& y_0 \geq \sum_{ip} -v_{ip}^e s_{ip} + \sum_{jkp} u_{jkp}^e r_{kp} y_{jk} & e = 1..E \\
& z,\ y \in \{0,1\}
\end{aligned} \tag{9.24}$$

Example

Consider the problem of designing the strategic production and distribution network. There are two products, two manufacturing plants and three customers. There are also two potential distribution center sites. The potential network is shown in Figure 13. A customer can only be served by a single warehouse. There is no minimum volume required to keep a warehouse open.

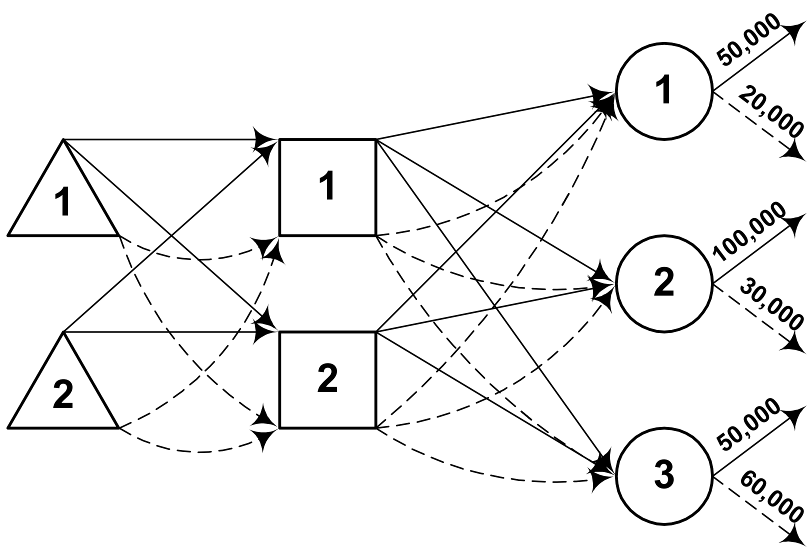


Figure 9.13. Potential Production-Distribution Network

Warehouse 1 has a fixed cost of \$100,000, an inventory carrying cost of \$0.5/cwt, a handling cost of \$2/cwt, and a handling capacity of 110,000 cwt. Warehouse 2 has a fixed cost of \$500,000, an inventory carrying cost of $1.5/cwt, a handling cost of $1/cwt, and a handling capacity of 500,000 cwt. Plant 1 has a production capacity of 60,000 cwt and 50,000 cwt for products 1 and 2, respectively, and production cost of $4/cwt and $3/cwt for products 1 and 2, respectively. Plant 2 has a production capacity of 400,000 cwt and 500,000 for products 1 and 2, respectively, and production cost of $5/cwt and $4/cwt for products 1 and 2, respectively. The demands for each product at the three customers are shown in the Figure 13, where the solid lines indicated product 1 and the dashed lines indicated product 2. All material flows are expressed in hundredweight (cwt). The transportation costs per hundredweight are given in the following table.

Table 9.3. Transportation Cost per Hundredweight (cwt)

Origin	Destination	Cost Product 1	Cost Product 2
P_1	W_1	0	0
P_1	W_2	5	5
P_2	W_1	4	4
P_2	W_2	2	2
W_1	C_1	4	3
W_1	C_2	3	2
W_1	C_3	5	4
W_2	C_1	2	3
W_2	C_2	1	2
W_2	C_3	2	3

Determine the optimal production-distribution system using the mixed integer programming solver of your choice. Show in an appendix to your report the formulation that you used, with all parameters and costs as numerical values.

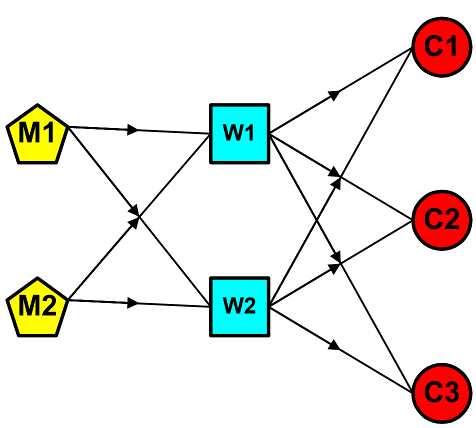


Figure 9.14. Arc-Based Single Echelon Supply Chain Example Illustration

Microsoft Excel - Supply Chain Design Ballou Example.xls

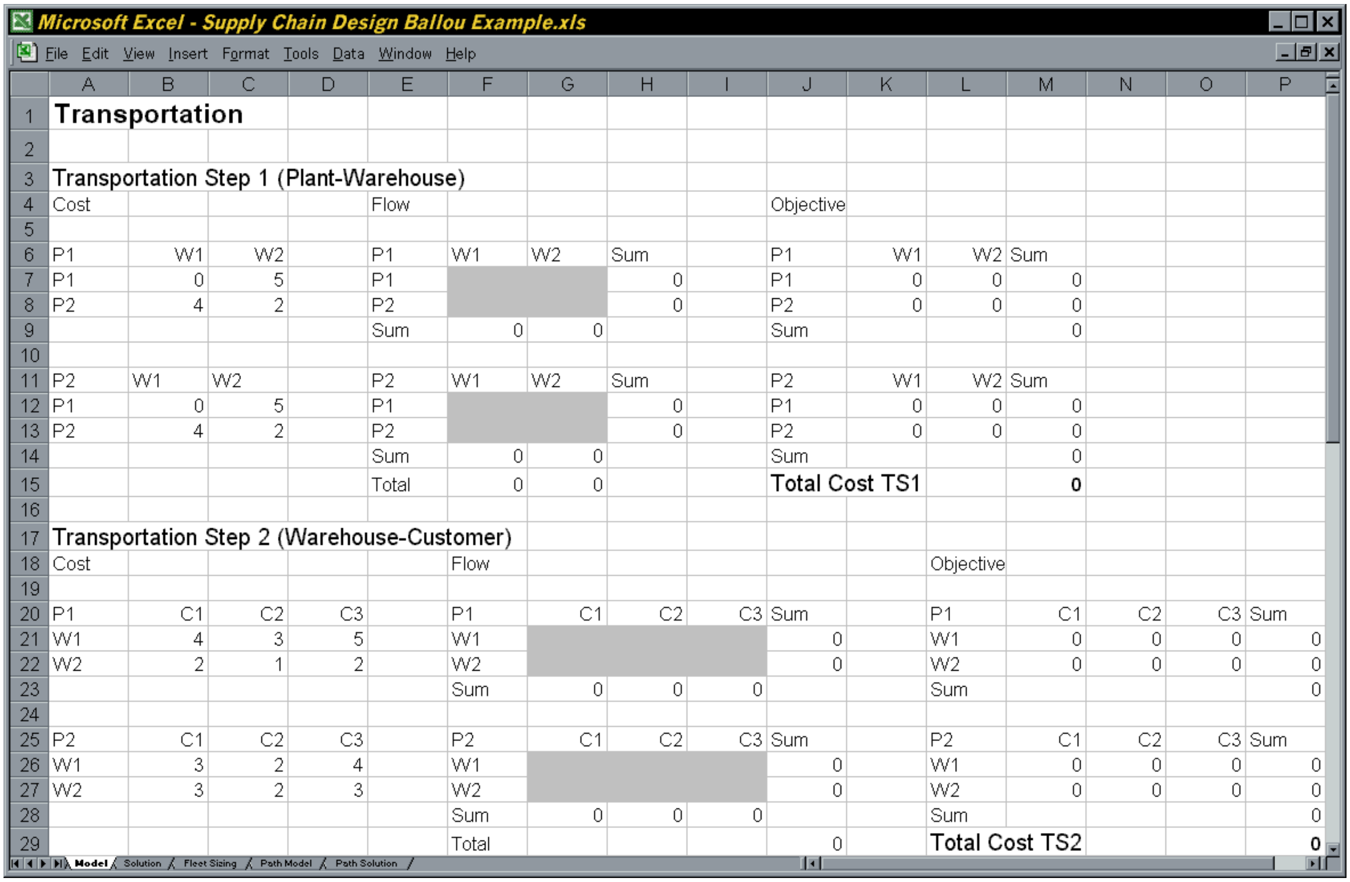
	A	B	C	D	E	F	G	H	I	J
1	Cost Data									
2										
3	Transportation Step 1 (Plant-Warehouse)					Transportation Step 2 (Warehouse-Customer				
4										
5		Cost	Destination				Cost	Destination		
6	Product	Origin	W1	W2		Product	Origin	C1	C2	C3
7	P1	P1	0	5		P1	W1	4	3	5
8		P2	4	2			W2	2	1	2
9										
10	P2	P1	0	5		P2	W1	3	2	4
11		P2	4	2			W2	3	2	3
12										
13	Production				Handling				Facility Costs	
14										
15	Product	Plant	Cost		Product	Warehous	Cost		Facility	Cost
16	P1	P1	4		P1	W1	2		W1	100000
17		P2	4			W2	1		W2	500000
18										
19	P2	P1	3		P2	W1	2			
20		P2	2			W2	1			

Figure 9.15. Supply Chain Ballou Example Transportation Data

Microsoft Excel - Supply Chain Design Ballou Example.xls

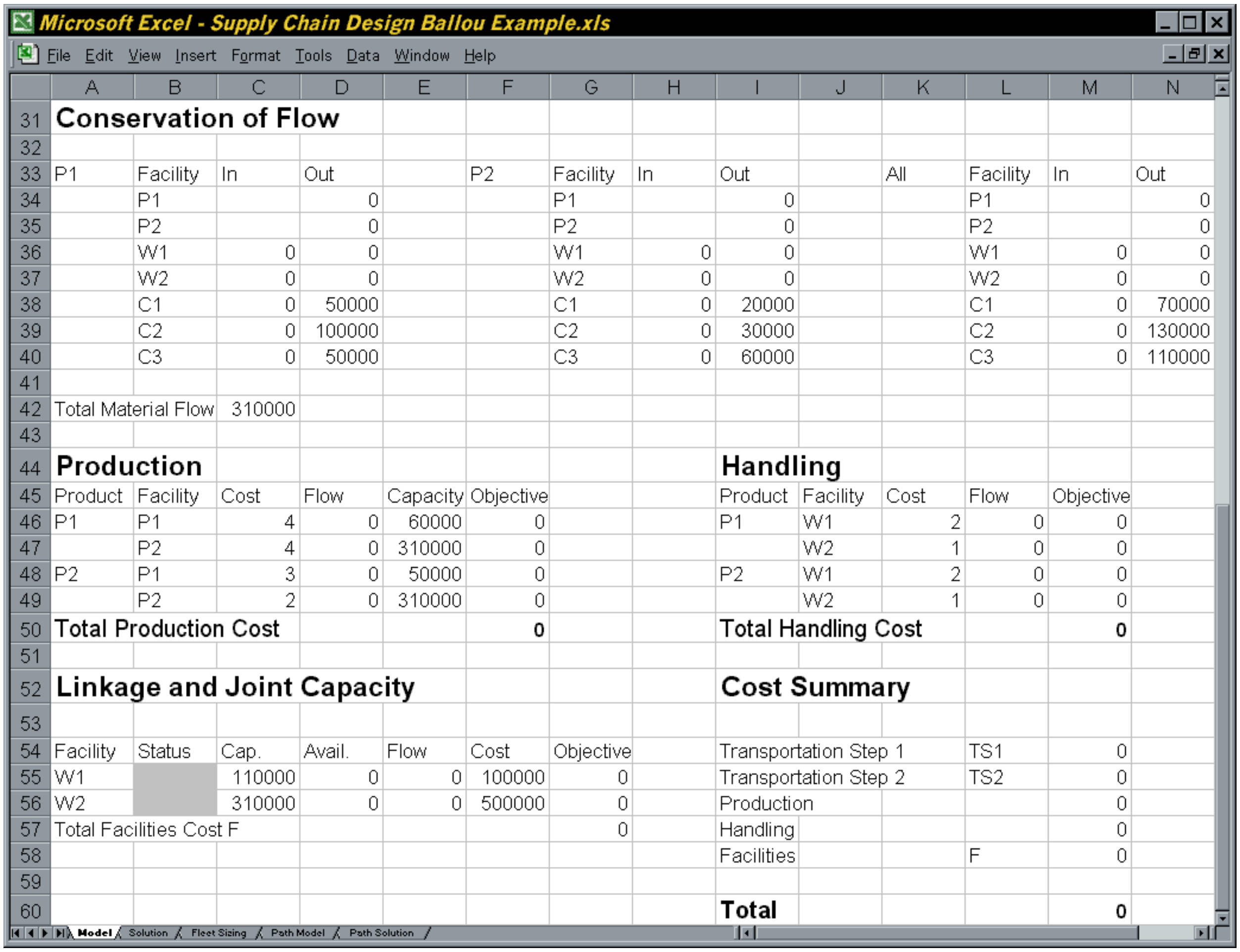
	A	B	C	D	E	F	G	H
22	Capacities							
23								
24	Warehouse Capacity (Throughput)					Production		
25								
26	Facility	Capacity				Product	Facility	Capacity
27	W1	110000				P1	P1	60000
28	W2						P2	
29						P2	P1	50000
30							P2	
31	Customer Demands							
32								
33	Product	C1	C2	C3				
34	P1	50000	100000	50000				
35	P2	20000	30000	60000				

Figure 9.16. Supply Chain Ballou Example Facility Capacity and Cost Data



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Transportation**															
2																
3	Transportation Step 1 (Plant-Warehouse)															
4	Cost				Flow					Objective						
5																
6	P1	W1	W2		P1	W1	W2	Sum		P1	W1	W2	Sum			
7	P1	0	5		P1			0		P1	0	0	0			
8	P2	4	2		P2			0		P2	0	0	0			
9					Sum	0	0			Sum			0			
10																
11	P2	W1	W2		P2	W1	W2	Sum		P2	W1	W2	Sum			
12	P1	0	5		P1			0		P1	0	0	0			
13	P2	4	2		P2			0		P2	0	0	0			
14					Sum	0	0			Sum			0			
15					Total	0	0			Total Cost TS1			**0**			
16																
17	Transportation Step 2 (Warehouse-Customer)															
18	Cost					Flow						Objective				
19																
20	P1	C1	C2	C3		P1	C1	C2	C3	Sum		P1	C1	C2	C3	Sum
21	W1	4	3	5		W1				0		W1	0	0	0	0
22	W2	2	1	2		W2				0		W2	0	0	0	0
23						Sum	0	0	0			Sum				0
24																
25	P2	C1	C2	C3		P2	C1	C2	C3	Sum		P2	C1	C2	C3	Sum
26	W1	3	2	4		W1				0		W1	0	0	0	0
27	W2	3	2	3		W2				0		W2	0	0	0	0
28						Sum	0	0	0			Sum				0
29						Total				0		Total Cost TS2				**0**

Figure 9.17. Arc-Based Supply Chain Ballou Example Transportation Model



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
31	**Conservation of Flow**													
32														
33	P1	Facility	In	Out		P2	Facility	In	Out		All	Facility	In	Out
34		P1		0			P1		0			P1		0
35		P2		0			P2		0			P2		0
36		W1	0	0			W1	0	0			W1	0	0
37		W2	0	0			W2	0	0			W2	0	0
38		C1	0	50000			C1	0	20000			C1	0	70000
39		C2	0	100000			C2	0	30000			C2	0	130000
40		C3	0	50000			C3	0	60000			C3	0	110000
41														
42	Total Material Flow		310000											
43														
44	**Production**								**Handling**					
45	Product	Facility	Cost	Flow	Capacity	Objective			Product	Facility	Cost	Flow	Objective	
46	P1	P1	4	0	60000	0			P1	W1	2	0	0	
47		P2	4	0	310000	0				W2	1	0	0	
48	P2	P1	3	0	50000	0			P2	W1	2	0	0	
49		P2	2	0	310000	0				W2	1	0	0	
50	Total Production Cost					**0**			Total Handling Cost				**0**	
51														
52	**Linkage and Joint Capacity**								**Cost Summary**					
53														
54	Facility	Status	Cap.	Avail.	Flow	Cost	Objective		Transportation Step 1			TS1	0	
55	W1		110000	0	0	100000	0		Transportation Step 2			TS2	0	
56	W2		310000	0	0	500000	0		Production				0	
57	Total Facilities Cost F						0		Handling				0	
58									Facilities			F	0	
59														
60									**Total**				**0**	

Figure 9.18. Arc-Based Supply Chain Ballou Example Facility Capacity Model

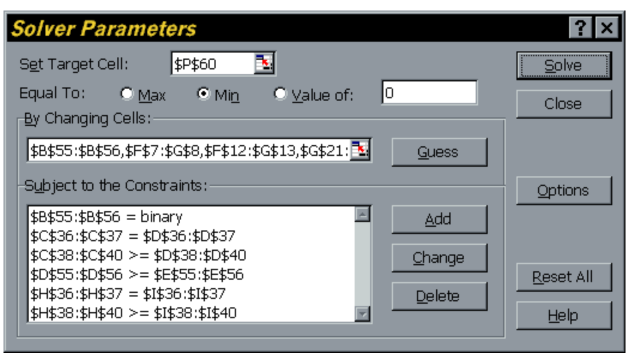


Figure 9.19. Arc-Based Supply Chain Ballou Example Solver

Microsoft Excel - Supply Chain Design Ballou Example.xls

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Transportation**															
2																
3	Transportation Step 1 (Plant-Warehouse)															
4	Cost				Flow					Objective						
5																
6	P1	W1	W2		P1	W1	W2	Sum		P1	W1	W2	Sum			
7	P1	0	5		P1	0	0	0		P1	0	0	0			
8	P2	4	2		P2	0	200000	200000		P2	0	400000	400000			
9					Sum	0	200000			Sum			400000			
10																
11	P2	W1	W2		P2	W1	W2	Sum		P2	W1	W2	Sum			
12	P1	0	5		P1	0	0	0		P1	0	0	0			
13	P2	4	2		P2	0	110000	110000		P2	0	220000	220000			
14					Sum	0	110000			Sum			220000			
15					Total	0	310000			Total Cost TS1			**620000**			
16																
17	Transportation Step 2 (Warehouse-Customer)															
18	Cost					Flow						Objective				
19																
20	P1	C1	C2	C3		P1	C1	C2	C3	Sum		P1	C1	C2	C3	Sum
21	W1	4	3	5		W1	0	0	0	0		W1	0	0	0	0
22	W2	2	1	2		W2	50000	100000	50000	200000		W2	100000	100000	100000	300000
23						Sum	50000	100000	50000			Sum				300000
24																
25	P2	C1	C2	C3		P2	C1	C2	C3	Sum		P2	C1	C2	C3	Sum
26	W1	3	2	4		W1	0	0	0	0		W1	0	0	0	0
27	W2	3	2	3		W2	20000	30000	60000	110000		W2	60000	60000	180000	300000
28						Sum	20000	30000	60000			Sum				300000
29						Total				310000		Total Cost TS2				**600000**

Model | Solution | Fleet Sizing | Path Model | Path Solution

Figure 9.20. Arc-Based Supply Chain Ballou Example Transportation Solution

Microsoft Excel - Supply Chain Design Ballou Example.xls

File Edit View Insert Format Tools Data Window Help

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
31	**Conservation of Flow**															
32																
33	P1	Facility	In	Out		P2	Facility	In	Out		All	Facility	In	Out		
34		P1		0			P1		0			P1		0		
35		P2		200000			P2		110000			P2		310000		
36		W1	0	0			W1	0	0			W1		0		
37		W2	200000	200000			W2	110000	110000			W2	310000	310000		
38		C1	50000	50000			C1	20000	20000			C1	70000	70000		
39		C2	100000	100000			C2	30000	30000			C2	130000	130000		
40		C3	50000	50000			C3	60000	60000			C3	110000	110000		
41																
42	Total Material Flow		310000													
43																
44	**Production**											**Handling**				
45	Product	Facility	Cost	Flow	Capacity	Objective						Product	Facility	Cost	Flow	Objective
46	P1	P1	4	0	60000	0						P1	W1	2	0	0
47		P2	4	200000	310000	800000							W2	1	200000	200000
48	P2	P1	3	0	50000	0						P2	W1	2	0	0
49		P2	2	110000	310000	220000							W2	1	110000	110000
50	Total Production Cost					**1020000**						Total Handling Cost				**310000**
51																
52	**Linkage and Joint Capacity**											**Cost Summary**				
53																
54	Facility	Status	Cap.	Avail.	Flow	Cost	Objective					Transportation Step 1			TS1	620000
55	W1	0	110000	0	0	100000	0					Transportation Step 2			TS2	600000
56	W2	1	310000	310000	310000	500000	500000					Production				1020000
57	Total Facilities Cost F						500000					Handling				310000
58												Facilities			F	500000
59																
60												**Total**				**3050000**

Model / Solution / Fleet Sizing / Path Model / Path Solution

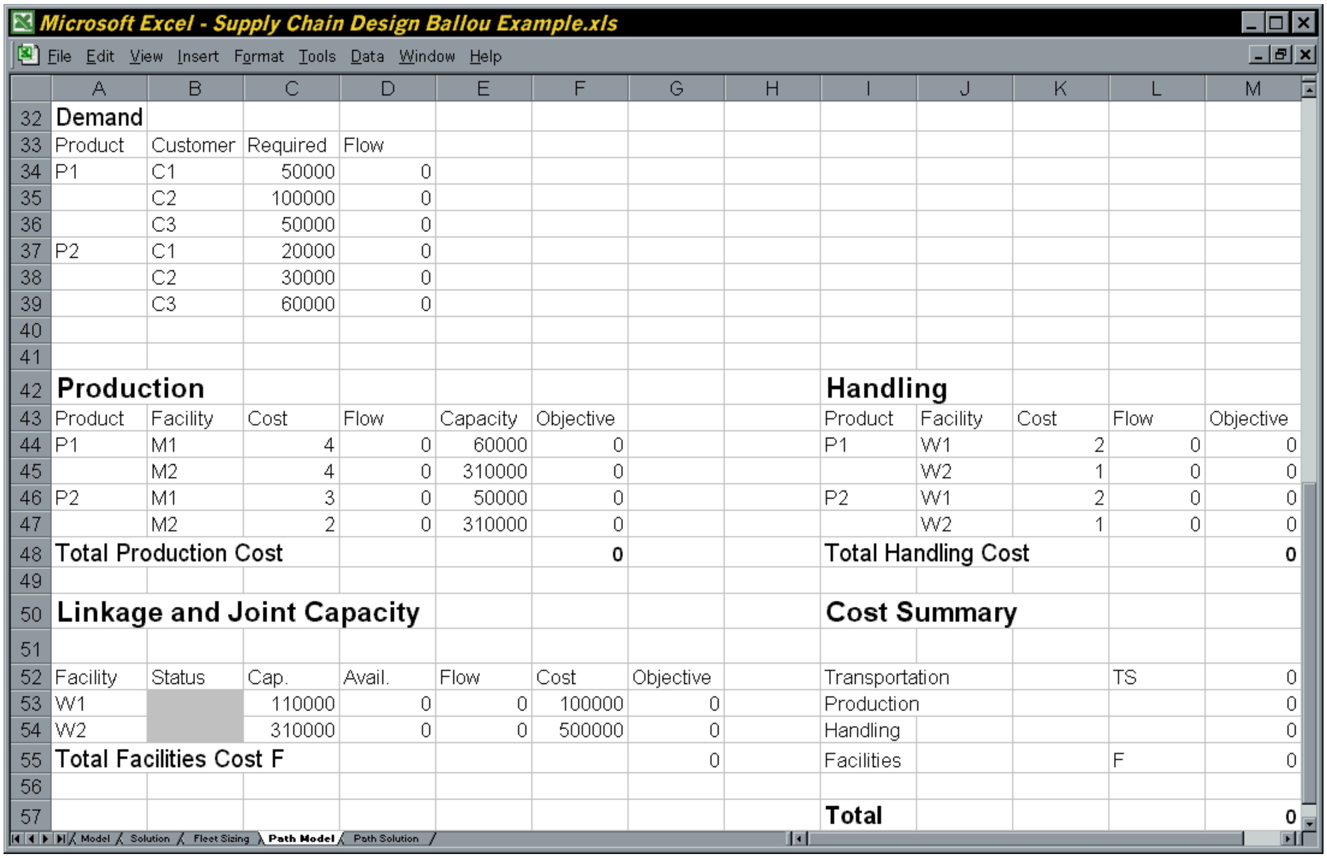
Figure 9.21. Arc-Based Supply Chain Ballou Example Facility Capacity Solution

Microsoft Excel - Supply Chain Design Ballou Example.xls

	A	B	C	D	E	F	G
1	**Transportation**						
2							
3	Flows						
4	Product	Plant	DC	Customer	Cost	Flow	Objective
5	P1	M1	W1	C1	4		0
6				C2	3		0
7				C3	5		0
8			W2	C1	9		0
9				C2	8		0
10				C3	10		0
11		M2	W1	C1	8		0
12				C2	7		0
13				C3	9		0
14			W2	C1	4		0
15				C2	3		0
16				C3	4		0
17	P2	M1	W1	C1	3		0
18				C2	2		0
19				C3	4		0
20			W2	C1	8		0
21				C2	7		0
22				C3	8		0
23		M2	W1	C1	7		0
24				C2	6		0
25				C3	8		0
26			W2	C1	5		0
27				C2	4		0
28				C3	5		0
29	Transportation Cost						**0**

Model / Solution / Fleet Sizing / Path Model / Path Solution

Figure 9.22. Path-Based Supply Chain Ballou Example Transportation Model



	A	B	C	D	E	F	G	H	I	J	K	L	M
32	Demand												
33	Product	Customer	Required	Flow									
34	P1	C1	50000	0									
35		C2	100000	0									
36		C3	50000	0									
37	P2	C1	20000	0									
38		C2	30000	0									
39		C3	60000	0									
40													
41													
42	Production								Handling				
43	Product	Facility	Cost	Flow	Capacity	Objective			Product	Facility	Cost	Flow	Objective
44	P1	M1	4	0	60000	0			P1	W1	2	0	0
45		M2	4	0	310000	0				W2	1	0	0
46	P2	M1	3	0	50000	0			P2	W1	2	0	0
47		M2	2	0	310000	0				W2	1	0	0
48	Total Production Cost					0			Total Handling Cost				0
49													
50	Linkage and Joint Capacity								Cost Summary				
51													
52	Facility	Status	Cap.	Avail.	Flow	Cost	Objective		Transportation			TS	0
53	W1		110000	0	0	100000	0		Production				0
54	W2		310000	0	0	500000	0		Handling				0
55	Total Facilities Cost F						0		Facilities			F	0
56													
57									Total				0

Figure 9.23. Path-Based Supply Chain Ballou Example Facility Capacity Model

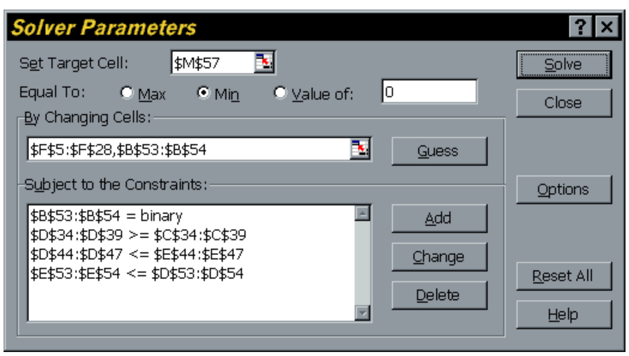


Figure 9.24. Path-Based Supply Chain Ballou Example Solver

Transportation

Flows

Product	Plant	DC	Customer	Cost	Flow	Objective
P1	M1	W1	C1	4	0	0
			C2	3	0	0
			C3	5	0	0
		W2	C1	7	0	0
			C2	6	0	0
			C3	7	0	0
	M2	W1	C1	8	0	0
			C2	7	0	0
			C3	9	0	0
		W2	C1	4	50000	200000
			C2	3	100000	300000
			C3	4	50000	200000
P2	M1	W1	C1	3	0	0
			C2	2	0	0
			C3	4	0	0
		W2	C1	8	0	0
			C2	7	0	0
			C3	8	0	0
	M2	W1	C1	7	0	0
			C2	6	0	0
			C3	8	0	0
		W2	C1	5	20000	100000
			C2	4	30000	120000
			C3	5	60000	300000
Transportation Cost						**1220000**

Figure 9.25. Path-Based Supply Chain Ballou Example Transportation Solution

Microsoft Excel - Supply Chain Design Ballou Example.xls

	A	B	C	D	E	F	G	H	I	J	K	L	M
32	**Demand**												
33	Product	Customer	Required	Flow									
34	P1	C1	50000	50000									
35		C2	100000	100000									
36		C3	50000	50000									
37	P2	C1	20000	20000									
38		C2	30000	30000									
39		C3	60000	60000									
40													
41													
42	**Production**								**Handling**				
43	Product	Facility	Cost	Flow	Capacity	Objective			Product	Facility	Cost	Flow	Objective
44	P1	M1	4	0	60000	0			P1	W1	2	0	0
45		M2	4	200000	310000	800000				W2	1	200000	200000
46	P2	M1	3	0	50000	0			P2	W1	2	0	0
47		M2	2	110000	310000	220000				W2	1	110000	110000
48	Total Production Cost					**1020000**			Total Handling Cost				**310000**
49													
50	**Linkage and Joint Capacity**								**Cost Summary**				
51													
52	Facility	Status	Cap.	Avail.	Flow	Cost	Objective		Transportation			TS	1220000
53	W1	0	110000	0	0	100000	0		Production				1020000
54	W2	1	310000	310000	310000	500000	500000		Handling				310000
55	Total Facilities Cost F						**500000**		Facilities			F	500000
56													
57									**Total**				**3050000**

Model / Solution / Fleet Sizing / Path Model / **Path Solution**

Figure 9.26. Path-Based Supply Chain Ballou Example Facility Capacity Solution

AMPL Model and Solution File

```
# Data for the mathematical model for Strategic Network Design, prepared by
# Edgar E. Blanco, Themis Rafailidis, Marcos Katsoulakis on June 2, 1997
#=========Set Definition =======
#--- commodities (index i)
set COMMODITY:=C1 C2;
#--- production plants
set PLANT:=P1 P2;
#--- warehouses
set WAREHOUSE:=W1 W2;
#--- customers
set CUSTOMER:=CU1 CU2 CU3;
#==========Constraint Paramenters ==========
param capacity:  P1    P2:=
       C1      60000   2000000
       C2      50000   2000000;
# Plant 2 has infinite capacity, but the total demand is never greater than 2000000 units!
param demand: CU1      CU2     CU3:=
       C1      50000   100000  50000
       C2      20000    30000  60000;
param min_ware :=
       W1      0
       W2      0;
param max_ware :=
       W1       110000
       W2       400001;
#==========Cost Paramenters ==========
param fix_ware:=
       W1      100000
       W2      500000;
param thr_ware:=
```

```
       W1     2
       W2     1;
param carr_ware:=
       W1     0.5
       W2     1.5;
param unit_cost:=
[C1,P1,*,*]:  CU1    CU2    CU3:=
       W1     8      7      9
       W2     11     10     11
[C2,P1,*,*]:  CU1    CU2    CU3:=
       W1     6      5      7
       W2     11     10     11
[C1,P2,*,*]:  CU1    CU2    CU3:=
       W1     13     12     14
       W2     9      8      9
[C2,P2,*,*]:  CU1    CU2    CU3:=
       W1     11     10     12
       W2     9      8      9;
#----Constraint Parameters
 param capacity{COMMODITY, PLANT} >=0;

#supply (production capacity) for commodity i at plant j
 param demand{COMMODITY, CUSTOMER} >=0;

#demand for commodity i in demand zone l
 param min_ware{WAREHOUSE} >=0;

#minimum allowed annual throughput for the warehouse k
 param max_ware{WAREHOUSE} >=0;

#maximum allowed annual throughput for the warehouse k
#---- Cost Parameters (annual based)
 param fix_ware{WAREHOUSE};

#fixed cost for opening and operating the warehouse k
 param thr_ware{WAREHOUSE};

#variable cost per unit of flow out of warehouse k
 param carr_ware{WAREHOUSE};

#inventory carrying cost per unit of flow out of warehouse k
 param unit_cost{COMMODITY,PLANT,WAREHOUSE,CUSTOMER};
# average unit cost of producing commodity i in plant j and
# shipping it through warehouse k to customer l
#----Variable Definition
 var flow{COMMODITY,PLANT,WAREHOUSE,CUSTOMER} >=0;
# amount of units of commodity i produced in plant j and
# shipped through warehouse k to customer l
 var serve_cust{WAREHOUSE,CUSTOMER} binary;
# 1 if warehouse k serves customer l, 0 otherwise
 var open_ware{WAREHOUSE} binary;
# 1 if warehous k is opened, 0 otherwise
#----Model description
#-----Objective Function
minimize total_cost:
       sum{i in COMMODITY, j in PLANT, k in WAREHOUSE, l in CUSTOMER} unit_cost[i,j,k,l]*flow[i,j,k,l]
       + sum{k in WAREHOUSE}
            ( fix_ware[k]*open_ware[k] +
              (thr_ware[k]+carr_ware[k])*(sum{l in CUSTOMER} (sum{i in COMMODITY}
demand[i,l])*serve_cust[k,l]) );
#----Constraints
#------Available production capacity
subject to Prod_Capacity{i in COMMODITY, j in PLANT}:
       sum{k in WAREHOUSE, l in CUSTOMER} flow[i,j,k,l] <= capacity[i,j];
#------All the demand must be met
subject to Satisfy_Demand{i in COMMODITY, k in WAREHOUSE, l in CUSTOMER}:
       sum{j in PLANT} flow[i,j,k,l] = demand[i,l]*serve_cust[k,l];
#-----Single customer assignment to warehouse
subject to Single_Customer{l in CUSTOMER}:
       sum{k in WAREHOUSE} serve_cust[k,l]=1;
#-----Keeps warehouse throughput in the limits
subject to Min_Flow{ k in WAREHOUSE}:
       sum{l in CUSTOMER} ( sum {i in COMMODITY} demand[i,l]) * serve_cust[k,l] >=
min_ware[k]*open_ware[k];
```

```
subject to Max_Flow{ k in WAREHOUSE}:
       sum{l in CUSTOMER} ( sum {i in COMMODITY} demand[i,l]) * serve_cust[k,l] <=
max_ware[k]*open_ware[k];
```

AIMMS Model and Solution File

```
! Strategic_Network_Design
! David J. Newton, Gina Robinson, Jeff Day
! May 28, 1997
! file SND.aim

SETS:
        Production_Plants,
        Warehouses,
        Customers,
        Products;

INDICES:
        i in Production_Plants,
        j in Warehouses,
        k in Customers,
        p in products;

PARAMETERS:
        carrying_costs(j) -> [0,inf]           "indictes the inventory carrying costs for warehouse j",
        Production_costs(i,p) -> [0,inf]       "the variable production costs per unit at plant i",
        Warehouse_capacity(j) -> [0,inf]       "the maximum capacity constraint for warehouse j",
        Production_capacity(i,p) -> [0,inf]    "the maximum capacity constraint for plant i",
        Warehouse_cost(j) -> [0,inf]           "fixed cost of warehouse j being open ",
        Demand(k,p) -> [0,inf]                 "customer demands from warehouse j for product p",
        Transportation_cost(i,j,p) -> [0,inf]   "cost to transport goods from plant i to warehouse j",
        Transportation_cost2(j,k,p) -> [0,inf]  "cost to transport goods from warehouse j to customer
k";

VARIABLES:
       x1(i,j,p)   -> [0,inf]       "flow between plant i and warehouse j",
       x2(j,k,p)   -> [0,inf]       "flow between warehouse j and customer k",
       y_customer(j,k) -> {0,1}     "1 if customer k is being served by warehouse j - 0 otherwise",
       y_warehouse(j) -> {0,1}      "1 if warehouse is to be open at location j - 0 otherwise",
       y_plant(i) -> {0,1}          "1 if warehouse is to be open at location i - 0 otherwise",
       Total_Cost;

CONSTRAINTS:
        Warehouse_Capacity_constraint(j)..
                Sum[(i,p), x1(i,j,p)] <= warehouse_capacity(j) * y_warehouse(j),

        Production_Capacity_constraint (i,p) ..
                Sum[(j), x1(i,j,p)] <= Production_capacity(i,p) * y_plant(i),

        Demand_Constraint (j,k,p) ..
                x2(j,k,p) = Demand(k,p)*y_customer(j,k),

        Balance_Constraint (j,p)..
                Sum[(i),x1(i,j,p)] = Sum[(k),x2(j,k,p)],

        Warehouse_Constraint(k)..
                Sum[(j), y_customer(j,k)] = 1,

        Total_Cost_Function ..
           Total_Cost = Sum[(i,j,p), x1(i,j,p)*(Production_costs(i,p)+Transportation_cost(i,j,p))] +
               Sum[(j), y_warehouse(j)*Warehouse_cost(j)] +
               Sum[(i,j,p), x1(i,j,p)*carrying_costs(j)] +
               Sum[(j,k,p), x2(j,k,p) * Transportation_cost2(j,k,p)];

MODEL:
                Strategic_Network_Design
                minimize : Total_Cost
                subject to: all
                method: mip;
```

```
! Data Section
$ include a:SND.dat

! Execution Section
SOLVE Strategic_Network_Design;
display Total_Cost, x1, x2, y_warehouse, y_plant, y_customer;

!file snd.dat
SETS:
         Production_Plants := {M1, M2},
         Warehouses        := {W1, W2},
         Customers         := {C1, C2, C3},
         Products          := {P1, P2};

COMPOSITE TABLE:
!Warehouse handling and carrying costs
          j               carrying_costs
!---------------------------------------
     W1                 2.5
     W2                 2.5;

COMPOSITE TABLE:
!Cost to produce each product at plant i
        i               p         production_costs
!----------------------------------------------------
     M1                 P1         4
     M1                 P2         3
     M2                 P1         5
     M2                 P2         4;

COMPOSITE TABLE:
!Capacity of Warehouse j
        j               Warehouse_capacity
!---------------------------------------------------
    W1                110000
    W2                500000;

COMPOSITE TABLE:
!Production capacity of each product at plant i
        i             p             Production_capacity
!---------------------------------------------------
     M1          P1             60000
     M1          P2             50000
     M2          P1            200000
     M2          P2            110000;

COMPOSITE TABLE:
!Table of fixed warehouse opening cost
        j               Warehouse_cost
!---------------------------------------------------
    W1                100000
    W2                500000;

COMPOSITE TABLE:
!Demand of each customer for each product
        k               p               Demand
!----------------------------------------------------
     C1          P1             50000
     C1          P2             20000
     C2          P1            100000
     C2          P2             30000
      C3         P1             50000
      C3         P2             60000;

COMPOSITE TABLE:
!Transportation costs from plant to warehouse
      i              j                p           Transportation_cost
!--------------------------------------------------------------------------
    M1       W1        P1                   0
    M1       W2        P1                   5
    M2       W1        P1                   4
```

```
    M2       W2        P1                  2
    M1       W1        P2                  0
    M1       W2        P2                  5
    M2       W1        P2                  4
    M2       W2        P2                  2;

COMPOSITE TABLE:
!Transportation costs from warehouse to customer
      j              k                  p          Transportation_cost2
!-----------------------------------------------------------------------
   W1       C1             P1                  4
   W2       C1             P1                  2
   W1       C2             P1                  3
   W2       C2             P1                  1
   W1       C3             P1                  5
   W2       C3             P1                  2
   W1       C1             P2                  3
   W2       C1             P2                  3
   W1       C2             P2                  2
   W2       C2             P2                  2
   W1       C3             P2                  4
   W2       C3             P2                  3;
```

9.6. Fixed Costs Calculations

One of the most difficult aspects of strategic distribution models is the determination of the proper fixed cost for large capital-intensive assets such as facilities, buildings, and major machining lines. A further complication is the interaction with corporate accounting and their rules and with the taxing authorities and their rules. While in many operational and tactical decision support systems the impact of taxes can be ignored, this is no longer the case for strategic decisions.

After-Tax Fixed Costs Calculations

Example

Assume a corporation has a constant marginal tax rate (*mtr*) of 40 %, a minimum attractive rate of return (*MARR*) of 15 %. The company is considering the purchase of a major manufacturing asset with a useful life (*L*) of seven years.

The first task is compute the net present value (*NPV*) of the cost and income flows associated with the purchase of the machine. The purchase price or initial investment cost is $2,000,000, the useful life (L) is seven years, there is no salvage value at the end of the seventh year, and the corporation is using a straight-line depreciation. Let cf_t denote the cash flow at the end of period *t*, where the cash flow is positive for revenue or income and is negative is expense or cost and time period zero denotes the current time. The net present value is then computed as:

$$NPV = \sum_{t=0}^{L} \frac{cf_t}{(1+MARR)^t} \tag{9.25}$$

$$\begin{aligned} NPV = {} & \frac{-2000000}{(1+0.15)^0} + \frac{285714}{(1+0.15)^1} + \frac{285714}{(1+0.15)^2} + \frac{285714}{(1+0.15)^3} + \\ & \frac{285714}{(1+0.15)^4} + \frac{285714}{(1+0.15)^5} + \frac{285714}{(1+0.15)^6} + \frac{285714}{(1+0.15)^7} \\ NPV = {} & -2000000 + 248477 + 216041 + 187862 + \\ & 163358 + 142050 + 123522 + 107411 = -811309 \end{aligned}$$

The Annual Equivalent Value (AEV) is then computed as follows:

$$AEV = NPV \cdot (A/P, MARR, L) \tag{9.26}$$

$$AEV = -811{,}309 \cdot 0.2404 = -195{,}039$$

It is the Annual Equivalent Value that is used as the fixed cost in strategic logistics systems design models, provided the basic time period in the model is a year.

The capital recovery factor (*crf*) is obtained by the dividing the annual equivalent value by the initial investment

$$crf = \frac{-195{,}039}{-2{,}000{,}000} = 0.0975$$

This capital recovery factor can now be used to obtain the after tax annual equivalent cost of any equipment or initial investment, provided the assumptions about the minimum attractive rate of return, the marginal tax rate, the investment life, the salvage value, and the depreciation schedule remain unchanged. A more extensive treatment of engineering economics for capital projects can be found in Park and Sharp (1990).

9.7. SMILE Integrated Distribution Model

Introduction

The following integrated distribution design model has been developed by Wei and Goetschalckx. The section comprises four subsections: verbal formulation, notation, detailed development, and mathematical formulation. Much of the model was developed jointly with other researchers in the Logistics Systems Program of the Material Handling Research Center, including M. Cole and K. Dogan.

Current Trends in Logistics Models and Algorithms

Computer Software Trends

- More capable MIP solvers
- Modeling languages
- ERP Systems (with some optimization)

Acceptance of Integrated Supply Chain View

- More comprehensive models (cradle to grave)
- More realistic models (inventory)

Models and Solution Algorithm Trends

- Models Growing in Complexity and Realism
- Models Developed by Supply Chain Owners
- Solution Techniques Become More Generic and "Shrink Wrapped"

Supply Chain Modeling and Algorithms Challenges

Multiple Periods

- Periodic demand
- Dynamic, multiperiod strategic systems

Global

- Taxes and profit realization
- Local contents, duty drawback

Stochastic

- Safety inventory
- Flexibility, robustness, risk, scenarios

Large Scale Models

Non-Linear Models

Stochastic Models

Standard MIP Linear Algorithms Cannot Solve Very Large Cases

NL-MIP or Stochastic Algorithms Only for Small Cases or Nonexistent

SMILE Production-Distribution Model

Model Characteristics

Multicommodity

Multi-echelon

Capacitated facilities

Capacitated channels

All costs

Deterministic

Single period

Single country or domestic

Arc formulation

Safety inventory proportional to demand (user determined)

Production cost and capacities

Weight and volume capacities

Depot and product single sourcing

Smart (tight) formulations

CPLEX MIP module

Verbal Formulation

Minimize total cost =

	warehouse inventory cost	(Z1)
+	warehouse assembly cost	(Z2)
+	warehouse facility cost	(Z3)
+	warehouse variable cost	(Z4)
+	plant facility cost	(Z5)
+	plant production cost	(Z6)
+	plant inventory cost	(Z7)
+	shipment cost	(Z8)
+	trunking inventory cost	(Z9)

Subject to:

flow conservation	(FC)
general flow conservation	(GF)
one warehouse type per site	(DT)
warehouse storage capacity	(ST, IS)
warehouse flow capacity	(TH, IF)
maximum/minimum open warehouses	(XD, ID)
plant either open or closed	(PO)
plant production capacity	(SP)
minimum/maximum number of carriers	(IC, XC)
carrier weight/volume capacity	(WE, VO)
ship to/from open warehouse only	(TD,FD)
ship from open plant only	(FP)

customer single sourcing	(SS1, 2, 3, 4)
max customer to warehouse distance	(MD)
max customer to warehouse time	(MT)
customer demand satisfaction	(DM, SDM)
restrictions on variable values	

Notation

For consistency, all time-based parameters and variables are described in terms of days. Another fundamental time period can be substituted everywhere for days.

Sets and Indexes

B	set of manufacturing facilities or plants
C	set of customers ($B \cap C = \emptyset$)
CS	set of customers with single sourcing requirements ($CS \subseteq C$)
D	set of warehouses or depots ($B \cap D = \emptyset$, $C \cap D = \emptyset$)
$L(j)$	set of warehouse types or sizes (e.g., small, medium, large, public) at warehouse site j ($j \in D$)
$M(i,j)$	set of parallel transportation channels to represent different transportation modes or cost structures (e.g., truckload, less-than-truckload, carload) from facility i to facility j ($i \in B \cup D$, $j \in D \cup C$), possibly empty set
P	set of products
PN	set of products not involved in assembly operations ($PN \subseteq P$)
PC	set of all products which can be assembly components for other products ($PC \subseteq P$)
PF	set of all final products of all the assembly operations ($PF \subseteq P$)
$PF(p)$	set of all final products which can include product p as an assembly component ($PF(p) \subseteq PF$)
i, j, k	index for plants, customers, or warehouses
l	index for warehouse types
m	index for transportation modes
p	index for products
f	index for final products

Parameters

ProdnCap_i	production capacity of plant i (resource units per day)
PlantFixedCost_i	fixed cost of plant i ($ per day)
$\text{PlantClosingCost}_i$	cost to close plant i, 0 unless the plant is currently open ($ per day)
$\text{PlantProdnCost}_{ip}$	production cost rate of plant i to produce one unit of product p ($ per unit of product p)
ProdnResUnit_{ip}	production resource units required for plant i to produce one unit of product p (resource units per unit of product p)
$\text{WhseFixedCost}_{jl}$	fixed cost of type l warehouse at site j ($ per day)
WhseClosingCost_j	cost to close an existing warehose at site j
$\text{WhseFlowCost}_{jlp}$	flow cost of product p through type l warehouse at site j ($ per unit flow of product p)
WhseFlowCap_{jl}	maximum resource capacity available at type l warehouse at site j (resource units per day)
FlowResUnit_{jlp}	flow resource units required for type l warehouse at site j to handle one unit of product p (resource units per unit of product p)
WhseStgCost_{jl}	storage capacity cost of type l warehouse at site j ($ per volume storage capacity per day)
WhseStgCap_{jl}	maximum storage capacity of type l warehouse at site j (volume unit storage)
SSFactor_{jp}	safety stock factor of product p at warehouse j
AssyCost_{jf}	cost to assemble one unit of final product f at warehouse j ($ per unit product f)
AssyAmt_{pf}	units of component product p required to assemble one unit of final product f
MaxWhseOpen	maximum allowable number of warehouses that can be open
MinWhseOpen	manimum number of warehouses required to be open
Demand_{kp}	demand of customer k for product p, possibly zero (unit product f per day)
MaxDistance_k	maximum allowable distance from customer k to its server warehouse (miles)
MaxTime_k	maximum allowable travel time to serve a customer k from its server warehouse (days)
Distance_{jkm}	travel distance on transportation channel jkm (miles)
$TransitTime_{ijm}$	travel time on transportation channel ijm (planning periods). This time interval is expressed in planning periods and thus this number is likely to be much smaller than 1. For example, if the planning period is one year and the travel time is a day, then $TransitTime = 0.0027397$.

R_{ijm}	time between shipments on transportation channel *ijm* (planning periods), i.e., order interval or replenishment interval. This time interval is expressed in planning periods and thus this number is likely to be much smaller than *1*. For example, if the planning period is one year and the order interval is a week, then $R = 0.01923$.
$CarrierCost_{ijm}$	transport carrier cost on channel *ijm* ($ per carrier shipment)
$TranUnitCost_{ijmp}$	transportation variable cost per unit of product *p* of transportation channel *ijm* ($ per unit *p* shipment)
$MinCarriers_{ijm}$	minimum required number of carriers per order interval on transportation channel *ijm* if the channel is used (carriers per order interval)
$MaxCarriers_{ijm}$	maximum allowable number of carriers per order interval on transportation channel *ijm* if the channel is used (carriers per order interval)
$CarrierWtCap_{ijm}$	carrier weight capacity of transportation channel *ijm* (weight units per carrier)
$CarrierVolCap_{ijm}$	carrier volume capacity of transportation channel *ijm* (volume units per carrier)
Value_p	value of a unit of product *p* ($ per unit of product *p*)
Wt_p	weight of a unit of product *p* (weight units per unit of product *p*)
Vol_p	volume of a unit of product *p* (volume units per unit of product *p*)
r	inventory carrying cost rate ($ per $ per day)

Decision Variables

x_{ijmp}	amount of product *p* shipped through transportation channel *ijm* (units of product *p* per planning period)
v_{jlp}	amount of product *p* flow through warehouse of type *l* at site *j* (units of product *p* per planning period)
u_{jlp}	maximum amount of product *p* stored at warehouse of type *l* at site *j* (units of product *p*)
w_{ijm}	number of carriers used per planning period from source facility *i* to destination facility *j* via transportation mode *m* (carriers per planning period)
s_i	(0,1) 1 if plant is opened at site *i*, 0 otherwise
sc_i	(0,1) 1 if plant is *not* opened at site *i*, 0 otherwise
y_{jl}	(0,1) 1 if type *l* warehouse is opened at site *j*, 0 otherwise
yc_j	(0,1) 1 if a warehouse is *not* opened at site *j*, 0 otherwise
z_{ijm}	(0,1) 1 if transportation channel *ijm* is opened, 0 otherwise

$q_{jk} \in \{0,1\}$ 1 if customer k is served from facility j, 0 otherwise

Detailed Development

This section contains a detailed mathematical development of the model. The notation is defined earlier in the notation section. Recall that all costs, time-based parameters, and variables are amortized and defined in terms of days to ensure that they are dimensionally consistent.

Products

The model can accommodate multiple products. Each product has a known weight, volume, and value that do not change during the course of distribution. Some products may not be produced at a plant or handled at a warehouse. If a plant or a warehouse can not process a product, the product flow from that plant or warehouse will be zero. For simplification, details are omitted from the model. In creating the formulation, e.g., the MPS file, it is easy to preprocess out those variables.

Warehousing

This section includes development of costs and constraints associated with warehouses.

Warehousing Inventory

This section develops the warehouse inventory cost function. It is assumed that every R days, each warehouse places an order for a product. The sketch below shows an idealized single product inventory cycle.

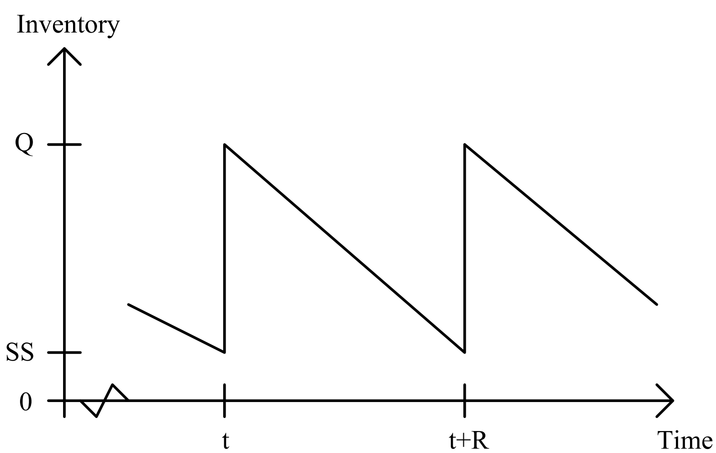


Figure 9.27. Idealized Single Product Inventory Cycle

Warehouse j placed an order and after the lead-time the order is received at time t. The inventory reach its highest level Q (Q = SS + demand rate × R) at time t. The cycle repeats after the next order is placed. The next order will be received at time t+R. Since the order interval is assumed to be the same as the shipping interval of the transportation channel from that source facility supplying the warehouse, R depends on the supply channel selected.

The warehouse inventory cost is simply the product of the inventory holding cost rate and the value of the average inventory. The mathematical form is developed below.

DemandRate_{jp} = demand rate for product p at warehouse j

Q_{jp} = expected maximum inventory of product p at warehouse j

SS_{jp} = safety stock of product p at warehouse j

Average Inventory_{jp} = average inventory of product p at warehouse j

$$\text{DemandRate}_{jp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} x_{ijmp}$$

For convenience of modeling, it's assumed that the worst possible case in which all product orders are replenished simultaneously.

$$Q_{jp} = SS_{jp} + Cycle\_Inventory_{jp}$$

$$Cycle\_Inventory_{jp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \mathrm{R}_{ijm}\, x_{ijmp}$$

Safety stock is the product flow times a preset safety stock factor, e.g., three days of product flow.

$$\text{SS}_{jp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \text{SSFactor}_{jp}\, x_{ijmp}$$

$$\text{Q}_{jp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \left(\text{SSFactor}_{jp} + \mathrm{R}_{ijm}\right) x_{ijmp}$$

$$Average\_Inventory_{jp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \left(\text{SSFactor}_{jp} + \frac{\mathrm{R}_{ijm}}{2}\right) x_{ijmp}$$

Thus, the warehouse inventory cost for warehouse j is

$$\sum_{p \in P} \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \mathrm{r}\ \text{Value}_p \left(\text{SSFactor}_{jp} + \frac{\mathrm{R}_{ijm}}{2}\right) x_{ijmp}$$

and the total system wide warehouse inventory cost is

$$\sum_{j \in D} \sum_{p \in P} \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \text{r Value}_p \left(\text{SSFactor}_{jp} + \frac{\text{R}_{ijm}}{2} \right) x_{ijmp} \tag{Z1}$$

Warehouse Assembly Operations

This section develops the model of the assembly operations at warehouses. Moiseenko (1984) presents a generalized multicommodity network flow model which allows conversion of different commodities. Still, commodity conversion can not model assembly operations; the product assembly is a more general problem.

For those products that are not involved in any assembly operations, constraint (FC) enforces conservation of flow at each warehouse.

$$\sum_{i \in B \cup D} \sum_{m \in M(i,j)} x_{ijmp} = \sum_{k \in C \cup D} \sum_{m \in M(j,k)} x_{jkmp} \quad j \in D, p \in PN \tag{FC}$$

However, flow conservation does not hold true for assembly operations. Consider the following example where two units of product *A* and three units of product *B* are combined to form one unit of product *C* and two units of product *B* and one unit of product *D* are combined to form one unit of product *E*. The typical assembly equations for the operations are:

$2\,A + 3\,B \rightarrow C$

$2\,B + 1\,D \rightarrow E$

In this case, the flow conservation condition must be replaced by some more general constraints. Let x denote the input flows and y denote the output flows. The general flow conservation equations for the warehouse assembly operations are

$$\begin{aligned} x_A - y_A &= 2(y_C - x_C) \\ x_B - y_B &= 3(y_C - x_C) + 2(y_E - x_E) \\ x_D - y_D &= y_E - x_E \end{aligned}$$

The general flow conservation conditions for warehouse assembly operations are as follows:

$$\begin{aligned} &\sum_{i \in B \cup D} \sum_{m \in M(i,j)} x_{ijmp} - \sum_{k \in C \cup D} \sum_{m \in M(j,k)} x_{jkmp} = \\ &\sum_{f \in PF(p)} \text{AssyAmt}_{pf} \left(\sum_{k \in C \cup D} \sum_{m \in M(j,k)} x_{jkmf} - \sum_{i \in B \cup D} \sum_{m \in M(i,j)} x_{ijmf} \right) \quad j \in D, p \in PC \end{aligned} \tag{GF}$$

The total warehouse assembly cost is

$$\sum_{j \in D} \sum_{f \in PF} \text{AssyCost}_{jf} \left(\sum_{k \in C \cup D} \sum_{m \in M(j,k)} x_{jkmf} - \sum_{i \in B \cup D} \sum_{m \in M(i,j)} x_{ijmf} \right) \qquad \text{(Z2)}$$

Warehouse Facility

The facility cost of each warehouse includes a cost proportional to storage capacity, a cost proportional to the amount shipped through the warehouse, and a fixed cost to either operate a warehouse or to close a warehouse. The fixed cost to operate a warehouse accounts for overhead, capital costs, and other costs that are not considered to be proportional to storage capacity or throughput. The cost to close an existing warehouse accounts for the various expenditures needed to cease operations at a particular site. Each of these costs is defined in terms of a warehouse site and a warehouse type. In general, each potential warehouse site can be occupied by one of several types of warehouses. For example, small, medium, and large may be three possible types of warehouse. The notion of multiple warehouse types, each with a linear cost structure, allows a nonlinear (but piecewise linear) cost structure for each potential warehouse site.

Each potential warehouse either be opened to exactly one type or be closed.

$$\sum_{l \in L(j)} y_{jl} + yc_j = 1 \qquad j \in D \qquad \text{(DT)}$$

The warehouse fixed and closing costs are expressed in (Z3)

$$\sum_{j \in D} \left(\sum_{l \in L(j)} \text{WhseFixedCost}_{jl} y_{jl} + \text{WhseClosingCost}_j \, yc_j \right) \qquad \text{(Z3)}$$

All products are assumed to be handled and stored in the same volume units, e.g., pallets, boxes, or trucks. All the warehouse capacities, warehouse storage costs, and warehouse handling costs are expressed in terms of the same volume units.

The worst case maximum amount of product p stored at warehouse of type l at site j, u_{jlp}, is defined by the constraints (ST) and (IS). The right hand side of constraint (IS), the worst case maximum inventory at the warehouse, is developed above in the section about warehouse inventory.

Constraint (ST) enforces the required warehouse storage capacity for each warehouse type.

$$\sum_{p \in P} \text{Vol}_p u_{jlp} \leq \text{WhseStgCap}_{jl} \, y_{jl} \quad j \in D, \; l \in L(j) \qquad \text{(ST)}$$

The amount of product p shipped through warehouse of type l at site j, v_{jlp}, is defined by the constraints (TH) and (IF). Constraint (IF) states that the amount shipped through a warehouse is equal to the product flow exists a warehouse.

$$\sum_{l \in L(j)} v_{jlp} = \sum_{k \in C \cup D} \; \sum_{m \in M(j,k)} x_{jkmp} \quad j \in D, p \in P \qquad \text{(IF)}$$

Constraint (TH) enforces that the flow through a warehouse cannot exceed the maximum handling capacity for each warehouse type. This constraint can also be used to model a general resource consumption proportional to the flow and resource capacity constraint.

$$\sum_{p \in P} \text{FlowResUnit}_{jlp} v_{jlp} \leq \text{WhseFlowCap}_{jl} \, y_{jl} \quad j \in D, \; l \in L(j) \qquad \text{(TH)}$$

The warehouse variable operating costs are expressed in (Z4). The second cost component is most of the times the warehouse handling cost, but can be any resource and cost proportional to the flow.

$$\sum_{j \in D} \sum_{l \in L(j)} \left(\text{WhseStgCost}_{jl} \sum_{p \in P} \text{Vol}_p u_{jlp} + \sum_{p \in P} \text{WhseFlowCost}_{jlp} v_{jlp} \right) \qquad \text{(Z4)}$$

There will be at most specified maximum allowable open warehouses and at least specified minimum required open warehouses.

Production

Plant costs include facility costs, production costs, and inventory costs. Plant facility costs comprise fixed costs to either operate a plant or to close a plant.

$$\sum_{i \in B} \left(\text{PlantFixedCost}_i s_i + \text{PlantClosingCost}_i sc_i \right) \qquad \text{(Z5)}$$

The following constraints enforce that each plant must be either open or closed.

$$s_i + sc_i = 1 \; i \in B \qquad \text{(PO)}$$

For each product, each plant is assumed to produce at a steady daily rate equal to the average number of units it ships per day. Plant production costs are incurred proportional to the number of units a product shipped out of a plant. The production cost per unit of product can differ between products and between plants.

$$\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\sum_{p\in P}\text{PlantProdnCost}_{ip}x_{ijmp} \tag{Z6}$$

Plant inventory costs are incurred only on finished goods inventories. Finished stocks are assumed to be accumulated at a steady rate. For each product and destination depot, the finished good inventory level at a plant follows the familiar saw tooth pattern. The total plant inventory cost is

$$\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\sum_{p\in P}\left(\frac{\text{r}}{2}\right)\text{Value}_p\ \text{R}_{ijm}\ x_{ijmp} \tag{Z7}$$

Plant production capacity is expressed in terms of Production Resource Units. Constraint (SP) prevents a plant from shipping products at a daily rate greater than its production capacity.

$$\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\sum_{p\in P}\text{ProdnResUnits}_{ip}\ x_{ijmp} \le \text{ProdnCap}_i s_i \quad i\in B \tag{SP}$$

Transportation

The transportation between facilities, or trunking, and local delivery to the final customers are assumed to be executed by direct shipment and vehicle routing is not considered. If routing cost estimates are available, then the local delivery costs can be based on these estimates. The total amount of product shipped during a planning period is assumed to be large compared to the size of a single carrier. Thus, integrality effects related to the number of carriers used can be ignored. Shipment costs comprise a variable cost proportional to the number of carriers used, the amount of weight units shipped, or both.

$$\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\left[\text{CarrierCost}_{ijm}w_{ijm}+\sum_{p\in P}\text{TranUnitCost}_{ijmp}x_{ijmp}\right] \tag{9.Z8}$$

Each unit shipped over channel *ijm* spends $TransitTime_{ijm}$ time in-transit from *i* to *j*. Inventory costs incurred in local delivery are ignored. Thus, the total transportation inventory costs are

$$\sum_{i\in B\cup D}\sum_{j\in D}\sum_{m\in M(i,j)}\sum_{p\in P} r\,Value_p\ TransitTime_{ijm}x_{ijmp} \tag{9.Z9}$$

Each transportation channel has a required minimum number of carriers used per order interval on that transportation channel if the transportation channel is used. The minimum number, which may be zero, is the enforced by constraint (IC).

$$R_{ijm} w_{ijm} \geq \text{MinCarriers}_{ijm} z_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{9.IC}$$

Each transportation channel has a limited maximum allowable number of carriers used per order interval. The maximum number, which may be infinity, is enforced by constraint (XC).

$$R_{ijm} w_{ijm} \leq \text{MaxCarriers}_{ijm} z_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{9.XC}$$

Each carrier has a weight capacity, enforced by constraint (WE),

$$\sum_{p \in P} \text{Wt}_p x_{ijmp} \leq \text{CarrierWtCap}_{ijm} w_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{9.WE}$$

Each carrier has a volume capacity, enforced by constraint (VO).

$$\sum_{p \in P} \text{Vol}_p x_{ijmp} \leq \text{CarrierVolCap}_{ijm} w_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{9.VO}$$

Constraints (TD) and (FD) require that transportation channels to and from a potential depot site are usable only if a depot is actually opened at that site.

$$z_{ijm} \leq \sum_{l \in L(j)} y_{jl} \qquad i \in B \cup D, j \in D, m \in M(i,j) \tag{9.TD}$$

$$z_{jkm} \leq \sum_{l \in L(j)} y_{jl} \qquad j \in D, k \in C \cup D, m \in M(j,k) \tag{9.FD}$$

Constraint (FP) requires that transportation channels from a plant are usable only if a plant is open.

$$z_{ijm} \leq s_i \qquad i \in B, j \in C \cup D, m \in M(i,j) \tag{9.FP}$$

Constraints (SS1) enforces that the customers requiring single sourcing be served by exactly one source.

$$\sum_{j \in B \cup D} q_{jk} = 1 \quad k \in CS \tag{9.SS1}$$

Constraints (SS2 and SS3) require that a customer can only be served from an open plant or an open warehouse.

$$q_{ik} \leq s_i \quad i \in B,\ k \in CS \tag{9.SS2}$$

$$q_{jk} \leq \sum_{l \in L(j)} y_{jl} \quad j \in D,\ k \in CS \tag{9.SS3}$$

Constraint (SS4) requires that transportation channels from a source to a customer are usable only if the customer is served by that source.

$$z_{jkm} \leq q_{jk} \qquad j \in B \cup D,\ k \in CS,\ m \in M(j,k) \tag{SS4}$$

Constraints (MD) and (MT) require that each customer be within a specified maximum travel distance and time from the warehouse or warehouse that serves it.

$$\text{Distance}_{jkm} z_{jkm} \leq \text{MaxDistance}_k \qquad k \in C, j \in B \cup D, m \in M\left(j,k\right) \tag{MD}$$

$$\text{TransitTime}_{jkm} z_{jkm} \leq \text{MaxTime}_k \qquad k \in C, j \in B \cup D, m \in M\left(j,k\right) \tag{MT}$$

Customers

Constraint (DM) ensures demand satisfaction. Constraints (DM) are for customers without single sourcing requirements.

$$\sum_{j \in B \cup D} \ \sum_{m \in M(j,k)} x_{jkmp} = \text{Demand}_{kp} \qquad k \in C, k \notin CS,\ p \in P \tag{DM}$$

Constraints (SDM) are for customers with single sourcing requirements.

$$\sum_{m \in M(j,k)} x_{jkmp} = q_{jk} \text{Demand}_{kp} \qquad j \in B \cup D, k \in CS, p \in P \tag{SDM}$$

Other customer-related constraints involve transportation; they are listed in section 4.3.3.

Decision Variable Values

The values obtained by the decision variables are constrained as follows.

$$v_{jlp} \geq 0 \quad j \in D, l \in L(j),\ p \in P$$

$$u_{jlp} \geq 0 \quad j \in D, l \in L(j),\ p \in P$$

$$x_{ijmp} \geq 0 \quad i \in B \cup D,\ j \in C \cup D, m \in M(i,j),\ p \in P$$

$$w_{ijm} \geq 0 \quad i \in B \cup D,\ j \in C \cup D, m \in M(i,j)$$

$$s_i \in \{0,1\} \; i \in B$$

$$sc_i \in \{0,1\} \; i \in B$$

$$y_{jl} \in \{0,1\} \;\; j \in D, \; l \in L(j)$$

$$yc_j \in \{0,1\} \;\; j \in D$$

$$z_{ijm} \in \{0,1\} \; i \in B \cup D, \; j \in C \cup D, m \in M(i,j)$$

$$q_{jk} \in \{0,1\} \;\; j \in B \cup D, \; k \in CS$$

Mathematical Formulation

Minimize

$$\sum_{j\in D}\sum_{p\in P}\sum_{i\in B\cup D}\sum_{m\in M(i,j)} \text{r Value}_p\left(\text{SSFactor}_{jp}+\frac{\text{R}_{ijm}}{2}\right)x_{ijmp} \tag{Z1}$$

$$\sum_{j\in D}\sum_{f\in PF}\text{AssyCost}_{jf}\left(\sum_{k\in C\cup D}\sum_{m\in M(j,k)}x_{jkmf}-\sum_{i\in B\cup D}\sum_{m\in M(i,j)}x_{ijmf}\right) \tag{Z2}$$

$$\sum_{j\in D}\left(\sum_{l\in L(j)}\text{WhseFixedCost}_{jl}\,y_{jl}+\text{WhseClosingCost}_j\;yc_j\right) \tag{Z3}$$

$$\sum_{j\in D}\sum_{l\in L(j)}\left(\text{WhseStgCost}_{jl}\sum_{p\in P}\text{Vol}_p u_{jlp}+\sum_{p\in P}\text{WhseFlowCost}_{jlp}v_{jlp}\right) \tag{Z4}$$

$$\sum_{i\in B}\left(\text{PlantFixedCost}_i s_i+\text{PlantClosingCost}_i sc_i\right) \tag{Z5}$$

$$\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\sum_{p\in P}\text{PlantProdnCost}_{ip}x_{ijmp} \tag{Z6}$$

$$\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\sum_{p\in P}\left(\frac{\text{r}}{2}\right)\text{Value}_p\;\text{R}_{ijm}\;x_{ijmp} \tag{Z7}$$

$$\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M(i,j)}\left[\left(\frac{\text{CarrierCost}_{ijm}}{\text{R}_{ijm}}\right)w_{ijm}+\sum_{p\in P}\text{TranUnitCost}_{ijmp}x_{ijmp}\right] \tag{9.Z8}$$

$$\sum_{i\in B\cup D}\sum_{j\in D}\sum_{m\in M(i,j)}\sum_{p\in P}\text{r Value}_p\;\text{TransitTime}_{ijm}x_{ijmp} \tag{Z9}$$

Subject to

$$\sum_{i\in B\cup D}\sum_{m\in M(i,j)}x_{ijmp}=\sum_{k\in C\cup D}\sum_{m\in M(j,k)}x_{jkmp}\quad j\in D,\,p\in PN \tag{FC}$$

$$\begin{aligned}&\sum_{i\in B\cup D}\sum_{m\in M(i,j)}x_{ijmp}-\sum_{k\in C\cup D}\sum_{m\in M(j,k)}x_{jkmp}=\\&\sum_{f\in PF(p)}\text{AssyAmt}_{pf}\left(\sum_{k\in C\cup D}\sum_{m\in M(j,k)}x_{jkmf}-\sum_{i\in B\cup D}\sum_{m\in M(i,j)}x_{ijmf}\right)\quad j\in D,\,p\in PC\end{aligned} \tag{GF}$$

$$\sum_{l \in L(j)} y_{jl} + yc_j = 1 \quad j \in D \tag{DT}$$

$$\sum_{l \in L(j)} u_{jlp} = \sum_{i \in B \cup D} \sum_{m \in M(i,j)} \left(\text{SSFactor}_{jp} + \text{R}_{ijm} \right) x_{ijmp} \quad j \in D,\, p \in P \tag{IS}$$

$$\sum_{p \in P} \text{Vol}_p u_{jlp} \le \text{WhseStgCap}_{jl}\, y_{jl} \quad j \in D,\ l \in L(j) \tag{ST}$$

$$\sum_{l \in L(j)} v_{jlp} = \sum_{k \in C \cup D} \sum_{m \in M(j,k)} x_{jkmp} \quad j \in D,\, p \in P \tag{IF}$$

$$\sum_{p \in P} \text{FlowResUnit}_{jlp} v_{jlp} \le \text{WhseFlowCap}_{jl}\, y_{jl} \quad j \in D,\ l \in L(j) \tag{TH}$$

$$\sum_{j \in D} \sum_{l \in L(j)} y_{jl} \le \text{MaxWhseOpen} \tag{XD}$$

$$\sum_{j \in D} \sum_{l \in L(j)} y_{jl} \ge \text{MinWhseOpen} \tag{ID}$$

$$s_i + sc_i = 1 \qquad i \in B \tag{PO}$$

$$\sum_{j \in C \cup D} \sum_{m \in M(i,j)} \sum_{p \in P} \text{ProdnResUnits}_{ip}\, x_{ijmp} \le \text{ProdnCap}_i s_i \qquad i \in B \tag{SP}$$

$$w_{ijm} \ge \text{MinCarriers}_{ijm} z_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{IC}$$

$$w_{ijm} \le \text{MaxCarriers}_{ijm} z_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{XC}$$

$$\sum_{p \in P} \text{R}_{ijm} \text{Wt}_p x_{ijmp} \le \text{CarrierWtCap}_{ijm} w_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{WE}$$

$$\sum_{p \in P} \text{R}_{ijm} \text{Vol}_p x_{ijmp} \le \text{CarrierVolCap}_{ijm} w_{ijm} \qquad i \in B \cup D, j \in C \cup D, m \in M(i,j) \tag{VO}$$

$$z_{ijm} \le \sum_{l \in L(j)} y_{jl} \qquad i \in B \cup D, j \in D, m \in M(i,j) \tag{TD}$$

$$z_{jkm} \le \sum_{l \in L(j)} y_{jl} \qquad j \in D, k \in C \cup D, m \in M(j,k) \tag{FD}$$

$$z_{ijm} \le s_i \qquad i \in B, j \in C \cup D, m \in M(i,j) \tag{FP}$$

$$\sum_{j \in B \cup D} q_{jk} = 1 \qquad k \in CS \tag{SS1}$$

$$q_{ik} \le s_i \qquad i \in B, k \in CS \qquad \text{(SS2)}$$

$$q_{jk} \le \sum_{l \in L(j)} y_{jl} \qquad j \in D, k \in CS \qquad \text{(SS3)}$$

$$z_{jkm} \le q_{jk} \qquad j \in B \cup D, k \in CS, m \in M(j,k) \qquad \text{(SS4)}$$

$$\text{Distance}_{jkm} z_{jkm} \le \text{MaxDistance}_k \qquad j \in B \cup D, k \in CS, m \in M(j,k) \qquad \text{(MD)}$$

$$\text{TransitTime}_{jkm} z_{jkm} \le \text{MaxTime}_k \qquad k \in C, j \in B \cup D, m \in M(j,k) \qquad \text{(MT)}$$

$$\sum_{j \in B \cup D} \sum_{m \in M(j,k)} x_{jkmp} = \text{Demand}_{kp} \qquad k \in C, k \notin CS, p \in P \qquad \text{(DM)}$$

$$\sum_{m \in M(j,k)} x_{jkmp} = q_{jk} \text{Demand}_{kp} \qquad j \in B \cup D, k \in CS, p \in P \qquad \text{(SDM)}$$

$v_{jlp} \ge 0 \; j \in D, l \in L(j), p \in P$

$u_{jlp} \ge 0 \; j \in D, l \in L(j), p \in P$

$x_{ijmp} \ge 0 \; i \in B \cup D, j \in C \cup D, m \in M(i,j), p \in P$

$w_{ijm} \ge 0 \; i \in B \cup D, j \in C \cup D, m \in M(i,j)$

$s_i \in \{0,1\} \; i \in B$

$sc_i \in \{0,1\} \; i \in B$

$y_{jl} \in \{0,1\} \; j \in D, l \in L(j)$

$yc_j \in \{0,1\} \; j \in D$

$z_{ijm} \in \{0,1\} \; i \in B \cup D, j \in C \cup D, m \in M(i,j)$

$q_{jk} \in \{0,1\} \; j \in B \cup D, k \in CS$

The above model belongs to the class of large scale mixed integer programming formulations. These models have been much more difficult to solve than linear or network flow programming problems of equivalent size.

Optimal Solution Algorithms

Preprocessing

Preprocessing is a technique that simplifies rows and columns by eliminating redundant rows and variables, substituting and fixing variables, and increasing lower and decreasing upper bounds on variables. Some general MIP preprocessing techniques are discussed in Savelsbergh (1992) and have been implemented in MINTO. Some general LP preprocessing techniques are have been implemented in CPLEX. The preprocessing approach presented here uses the logistics problem information that can be easily identified and implemented from the problem data.

For several of the constraints, coefficients and right-hand sides can be tightened with little effort. Define *C*(*j*) as the set of customers which can be reached from facility (plant or warehouse) *j*, either directly or indirectly. Since the throughput of a facility can not exceed the total demand of all customers the facility can reach, constraint (SP) and (TH) can be adjusted as follows.

$$\sum_{j \in D \cup C} \sum_{m \in M} \sum_{p \in P} PPR_{ip} x_{ijmp} \le \min \left\{ \sum_{p \in P} \sum_{k \in C(i)} PPR_{ip} DM_{kp}, PPCap_i \right\} s_i \quad i \in B \qquad \text{(SP)}$$

$$\sum_{p \in P} WHR_{jlp} v_{jlp} \le \min \left\{ \sum \sum WHR_{jlp} DM_{kp}, WHCap_{jl} \right\} y_{jl} \quad j \in D, l \in L \qquad \text{(TH)}$$

Constraints (MD) an (MT) can be processed out by eliminating those channels that violate the maximum travel distance limit and the maximum travel time limit. Many cost components and constraints are optional depending on the problem instance. These can be ignored without affecting the solution. For example, all variables and constraints related to single sourcing, (q_{ik}) and (SS1)-(SS4), respectively, can be eliminated if a global status variable indicates that no single sourcing is required.

Valid Inequalities

For a particular product, the flow from a facility to a customer, who is directly supplied by this facility, can not exceed the demand of that customer, The following first two sets of valid inequalities can then be added to the constraint set to tighten the LP relaxation, corresponding to the cases where the supplying facility is a depot or a plant, respectively. For a particular product and if a channel requires a minimum number of carriers when the channel is used, then the flow for a particular product on that channel cannot exceed the demand of that customer. The third set of valid inequalities (VI3) can then be added to the constraint set to tighten the LP relaxation and it is valid for both plants and depots directly supplying that customer.

$$\sum_{m \in M} x_{jkmp} \leq Demand_{kp} \sum_{l \in L} y_{jl} \qquad j \in D, k \in C, p \in P \qquad \text{(VI1)}$$

$$\sum_{m \in M} x_{ikmp} \leq Demand_{kp} s_i \qquad i \in B, k \in C, p \in P \qquad \text{(VI2)}$$

$$x_{ikmp} \leq Demand_{kp} z_{ikm} \qquad i \in B \cup D, k \in C, m \in M, p \in P \qquad \text{(VI3)}$$

These inequalities are very similar to the Strong Linear Programming Relaxation (SLPR) of the Uncapacitated Facility Location problem (UFL). However, because the SLPR has a very large number of constraints, it is not trivial to solve SLPR even once for the root node of the branch-and-bound tree.

We have found that just a small fraction of the total number of (VI1) and (VI2) constraints contributes to a tighter relaxation. Since adding the (VI1) and (VI2) constraints will force the depot or plant open, it seems logical to add those constraints corresponding to the customer demands that are more important than others for a particular depot or plant. We selected the customers which are the closest to the depot or plant and the customers that have the largest demand expressed in weight units. Also since the transportation cost often is weight based, we selected first the (VI3) constraints for which the customer product demands have higher total weight.

Adding only a fraction of the total number of valid inequalities has proven to be an efficient solution strategy. Adding all valid inequalities created a linear relaxation that could not be solved in several instances because of memory constraints or solving such a large linear relaxation required excessive computer times. On the other hand, a branch-and-cut solution strategy successively generated valid but irrelevant inequalities and its total solution time also exceeded the time required by adding a fraction of the inequalities at the root node.

Branching Order

Finally, there exists a natural hierarchy in the design decisions for configuring production-distribution networks. The decision involving the opening or closing of plants have the most far reaching impact. Next, are the decisions regarding the establishment of a distribution center at a particular site. Once a site has been selected, then the best type of distribution center has be chosen. Finally, after all facility decisions have been made, then the transportation channels are selected and the (integer) number of carriers is determined for each channel. This hierarchy is given to the mixed integer programming solver in the form of a variable branching order. Again, the impact of selecting the wrong branching order can be very large, as illustrated in the RECYCLE case study.

Case Studies

The CIMPEL model and design environment were developed based on numerous real world case problems. The performance of CIMPEL in sample of these cases will be briefly reviewed.

Electronics Manufacturing (ELEC)

The design problem was to decide the number, location and size of distribution centers for an electronics manufacturer situated in the eastern and central section of the United States. There were 30 manufacturing plants, 30 different commodities, 98 customer facilities, 50 potential warehousing sites each with 6 different types, 6400 transportation channels each with a single transportation mode. All thirty products have weights equal to one and volumes equal to one. Each product can only be produced at one specific plant and that plant can only produce that product.. Plant production is uncapacitated and production cost was not modeled. Each warehouse has inbound channels from the plants and outbound channels to customers. There are no channels between warehouses and no channels between plants and customers, i.e., the distribution system consists of exactly one level of warehousing. All potential warehouse sites are identical, except in their location. Each warehouse site can have one of six different types, each with a fixed cost, handling cost, and handling capacity. The largest warehouse type can accommodate all the customer demands. Those six types represent a concave piecewise linear cost structure for each potential warehouse site. Warehouse storage is uncapacitated and storage cost was not modeled. There is no closing cost and no inventory cost associated with warehousing. Transportation cost is on a per unit basis. No customer requires single sourcing. Every customer requires the warehouse that serves it to be within 1300 miles travel distance. Customer demands are deterministic and exhibit a very wide range for different customers and products. The optimal configuration has only one potential warehouse open to largest type which serves all the customers. The solution times for the different solution methods are given in the next table.

Table 9.4. Solution Statistics for the ELEC Case

Problem	Solved	# Variables	# Binary Vars.	# Constraints	Solution Time (s)	# of Nodes Evaluated	LP Relax. Gap 100(Z -ZLP)/Z %
Original	No	119,858	350	5,555	> 20,072.66	1,482	15.48%
Strong LP Relaxation	No	119,858	350	123,563	> 54.76	0	Unknown
10% Strong LP Relax.	Yes	119,858	350	25,428	2413.91	0	0.00%

Clearly the original linear programming relaxation was to weak, so too many nodes had to be evaluated. On the other hand, the strong LP relaxation created such a large linear program that it could not be solved in the memory available (256 megabytes of core RAM). The LP relaxation with 10 % of the strong constraints could however solve the problem in the allotted time and memory.

The effect of tightening constraint (TH) was tested on a test case which is a variation of ELEC with 25 potential warehouses, 55 customers, and all other characteristics remain unchanged. The case was solved using CPLEX 2.1 on BERNINI (Pentium 90 PC). The solution times for the different solution methods are given in the next table.

Table 9.5. Solution Statistics for the ELEC Case with Preprocessing

	Solution Time (seconds)	Number of Nodes Evaluated	LP Relaxation Gap (Z -ZLP)/Z (%)
Before Tightening	5177.81	483	24.17
After Tightening	838.56	194	8.79

Wholesaler (HAC)

The design problem was to decide the number, location and size of distribution centers for a wholesale distributor of heating and air conditioning products. There were 18 manufacturing plants, 18 different commodities, 242 customer facilities, 21 potential warehousing sites each with one type, 5460 transportation channels each with a single transportation mode. Plant production is uncapacitated and production cost was not modeled. Each warehouse has inbound channels from the plants and outbound channels to customers. There are no channels between warehouses and no channels between plants and customers, i.e., the distribution system consists of exactly one level of warehousing. Potential warehouse sites have different costs. Warehouse storage is uncapacitated and storage cost was not modeled. There is no closing cost and no inventory cost associated with warehousing. Transportation cost is on a per unit basis. No customer requires single sourcing. Customer demands are deterministic and exhibit a very wide range for different customers and products. The optimal configuration has eight potential warehouse open. The solution times for the different solution methods are given in the next table.

Table 9.6. Solution Statistics for the HAC Case

Problem	Solved	# Variables	# Binary Vars.	# Constraints	Solution Time (s)	# of Nodes Evaluated	LP Relax. Gap 100(Z -ZLP)/Z %
Original	No	83,559	42	4,739	> 13,765	2,285	37.38%
Strong LP Relaxation	Yes	83,559	42	82,878	485	0	0.00%
10% Strong LP Relax.	Yes	83,559	42	21,665	192	23	0.24%

Clearly the original linear programming relaxation was to weak, so too many nodes had to be evaluated. On the other hand, the strong LP relaxation created such a large LP that it took longer to solve the overall problem even though the problem was solved at the root node. The LP relaxation with 20 % of the strong constraints could however solve the problem in a smaller amount of time. Observe that with one fourth of the total number of constraints the gap at the root node was less than a quarter of a percent.

Recycling System (RECYCLE)

The problem was to determine the location, number, and size of material recovery facilities in a three county area for the collection and processing of recyclable materials. The four materials in this problem were the materials most often collected during residential curb side collection: newspaper, glass, aluminum, and plastic. These materials have to be sorted, potentially cleaned, and baled for final transportation to various reclamation facilities. Since this is a recycling case, the real materials flow from the customers, through the material recovery facilities, to the reclamation plants. In the model all flows were still assumed to flow from plants, through depots, to customers. There were 5 reclamation plants, 4 commodities, 9 customers, 3 material recovery facilities, each with three different sizes, and 48 transportation channels. There was only one transportation mode, the collection trucks. There are no channels between warehouses and no channels between plants and customers, i.e., the distribution system consists of exactly one level of warehousing. Potential warehouse sites have different costs. Warehouse storage is uncapacitated and storage cost was not modeled. There is no closing cost and no inventory cost associated with warehousing. . No customer requires single sourcing. Customer demands are deterministic and exhibit a very wide range for different customers and products.

Because the collection trucks were very underutilized, a linear approximation of the number of carriers on a transportation channel was no longer valid, i.e., the number of carriers is required to be an integer number. The following branch order is used: first the solver branches on the depot binary variables and then on the integer carrier variables. The solution times for the different solution methods are given in the next table.

Table 9.7. Solution Statistics for the RECYCLE Case

Problem	Solved	# Variables	# Binary / Int. Vars.	# Constraints	Solution Time (s)	# of Nodes Evaluated	LP Relax. Gap 100(Z -ZLP)/Z %
Original	No	273	12 & 45	209	> 55,140	333,300	76.49%
Braching Order	Yes	273	12 & 45	209	2.63	147	76.49%

Packaging Operations (PACKAGE)

This problem involved the strategic planning of the whole supply chain of a packaging company. In a packaging operation paper roll stock from the paper mill is transformed into finished goods in two major stages of manufacturing, called plants and finishing facilities, for which the machines are called presses and gluers, respectively. The decisions included the supply mixes of the paper mills, allocation of the manufacturing resources (i.e. presses and gluers) to major product groups, and the configuration of the distribution system. The system included 2 paper mills, 9 potential locations for manufacturing and warehousing operations, 24 types of machines to conduct these manufacturing operations, and 250 major customer locations. The machines and the facilities each had a fixed cost and a variable production or handling cost. The machines, and some of the warehousing facilities had capacities. The speed of machines varied depending on the product and the location. The customers had known demands. The customer demands had a strong seasonal pattern. In an initial and traditional approach, we conducted the study for the peak demand period.

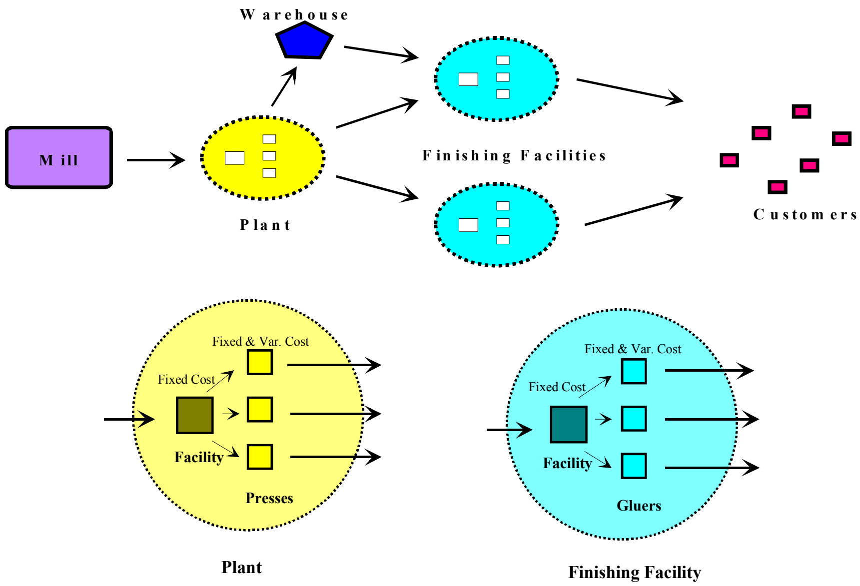


Figure 9.28. Schematic of a Two Stage Manufacturing System including Machines

The model structure allowed us to represent each of the machines, be it either a press or a gluer, as a distribution facility, while the facilities that housed the manufacturing resources also were modeled as warehouses. Transportation channels assured that no machine was used unless the facility that housed this machine was also used. The model described above was flexible and generic enough so that it could represent this production and distribution network including the machines inside the facilities for which was not initially designed. In the preceding figure the supply chain of the company is illustrated. The implementation of 100% strong formulation allowed the system to be solved to optimality.

Table 9.8. Solution Statistics for the PACKAGE Case

Problem	Solved	# of Variables	# of Binary Variables	# of Constraints	Solution Time (seconds)	# of Nodes evaluated	LP Relax. Gap 100*(Z-ZLP)/Z %
100% Strong	Yes	6446	317	4279	251	591	0.05%

Clearly, the fine tuning of the mixed integer programming formulation and solution algorithms has reduced dramatically the time required to solve the problems and has extended the size of the problems can be solved.

9.8. Conclusions

Without doubt, there will be design problems where our model is not sufficiently accurate or detailed. A prominent example is our assumption that the production cost and resource requirements for producing a particular product at a manufacturing plant are independent of the production quantity or the product mix produced at this plant. Also, our treatment of safety inventory is very simplistic but realistic. International considerations, such as local content, duties, and taxes are also not currently included in our system. At the same time, the execution times of our algorithms will be larger than that of specialized models and solution techniques when designing systems of comparable size.

But, it was our goal to reduce the time required to design a production and distribution system for a company without much prior experience in this area. The predefined structure of the model allows the company to focus on the data collection and sensitivity analysis parts of the design task. The algorithm enhancements allow SMILE to be solved for these real life cases in a fraction of the time of a general purpose solver, if the general purpose solver can reach a solution at all. At the same time, the CIMPEL modeling environment has been of significant help in communicating our models and algorithms with industrial companies and with students in our undergraduate, graduate and continuing education classes. We therefore believe there is a place for generic but domain specific design models, algorithms, and modeling environments that bridge the gap between multipurpose algebraic modeling languages and one-of-a-kind specialized models and associated solution algorithms.

Exercises

True-False Questions

An advantage of the path-based formulation over the arc-based formulation is that addition an additional echelon between source and sink does not dramatically increase the number of required variables, (T/F) _____(1).

For the design of strategic logistics systems the site generating algorithms are especially good at incorporating site dependent costs, (T/F)______(2),
while the site selecting algorithms are primarily strong at trading off fixed versus variable costs, (T/F)______(3).

In the Add heuristic for the discrete warehouse location problem, the warehouse or depot with the most negative site dependent cost is evaluated next for possible establishment (addition), (T/F)______(4).

It is the recommended practice that the first supply chain design project in a corporation should be for the strategic design of the supply chain, (T/F) _____(5).

The Geoffrion and Graves model for strategic logistics systems design enforces customer single sourcing service constraints, (T/F)______(6),
and is an arc-based formation, (T/F)______(7).

The Kuehn and Hamburger model for strategic logistics systems design enforces customer single sourcing service constraints, (T/F)______(8).

ColorJet Company

The ColorJet Company manufactures color inkjet printers. The manufacturing plant is located on the West Coast of the United States and the company operates a regional distribution center on the East Coast. You are responsible for selecting the least cost transportation mode between the manufacturing plant and the distribution center. The possible transportation modes are rail, piggyback (truck on rail), truck, and air. Various characteristics for each mode are given in the next table.

Table 9.9. Transportation Mode Selection Data

Characteristic	Rail	Piggyback	Truck	Air
Unit Transportation Cost ($)	1	1.5	2	14
Channel Transit Time (days)	21	14	5	1
Transportation Batch Size	1500	360	360	50

The unit manufacturing cost of the printers is $300. The aggregate holding cost for all inventories for one year is 30 % of the product value, i.e., 30 cents per dollar per year. The total customer demand served from the distribution center is 18,000 printers per year. It is assumed that both production and demand processes have a constant rate throughout the year. A year is equivalent to 365 days. The daily demand for printers at the distribution center has a coefficient of variation equal to two. The safety inventory for each mode in the distribution center is sufficient so that the probability of stock-out during the lead-time is less than 5 % for that transportation mode. The annualized fixed warehouse cost is equal to $250 per storage location and each storage location can hold 10 printers.

Determine the best transportation mode based on the total cost for this production-distribution system. Show your results in a clear table (alternatives versus costs) and compute all costs on an annual basis. Be sure to indicate the units for all numerical results.

Million Bubbles Company

The Million Bubbles Company manufactures Jacuzzi baths for customers in the continental United States and Alaska. The manufacturing plant is located in Macon, Georgia, on the East Coast of the United States and the company operates a regional distribution center in Seattle, Washington on the West Coast of the United States. You are responsible for selecting the least cost transportation mode between the manufacturing plant and the distribution center. The possible transportation modes are rail or truck. Various characteristics for each mode are given in the next table.

Table 9.10. Transportation Mode Selection Data

Characteristic	Rail	Truck
Unit Transportation Cost ($)	18	160
Channel Transit Time (days)	15	5
Transportation Batch Size	20	6

The unit manufacturing cost of the Jacuzzi baths is $1,500. Due to the annual style and color changes in the Jacuzzi models, the aggregate holding cost for all inventories for one year is 90 % of the product value, i.e., 90 cents per dollar per year. The total customer demand served from the distribution center is 360 Jacuzzi baths per year. It is assumed that both production and demand processes have a constant average rate throughout the year. A year is equivalent to 360 days. The daily demand for baths at the distribution center has a normal distribution with a mean of one bath and a standard deviation of 0.33 baths. The safety inventory for each mode in the distribution center is assumed to ensure a service level equivalent to a probability of 99.5 % delivery out of inventory for the daily demand of Jacuzzi baths. The annualized fixed warehouse cost is equal to $125 per bath.

Determine the best transportation mode based on the total cost for this production-distribution system. Show your results in a clear table (alternatives versus costs) and compute all costs on an annual basis. Be sure to indicate the units for all numerical results.

Smile Extension Assembly Operations in Warehouses

Consider the SMILE model by Wei and Goetschalckx for the design of strategic production-distribution systems. Assume that some assembly operations can be performed in the last warehouse before the products reach the customers. There are two intermediate products, denoted by A and B, and there are two final products, denoted by C and D. The assembly equations for the final assembly are:

$$\begin{cases} 2A + 3B = C \\ 2B + 5A = 2D \end{cases}$$

In other words, the first assembly equation states that two units of product A plus three units of product B are required to make one unit of product C.

Assembling one product C takes 9 minutes, assembling two units of product D takes 24 minutes. The handling of one unit of product C from receiving through shipping takes 3 minutes. The handling of one unit of product D from receiving through shipping takes 7 minutes. The handling of an assembled unit of product C takes 1 minute. The handling of an assembled unit of product D takes 2 minutes. All assembly and handling has to be done by a team of 10 equivalent workers. Each works 480 minutes per shift. The cost per minute of labor spent on either handling or assembling products C and D is equal to \$1. Assembling product D requires the presence of a particular machine. This presence is denoted by variable $Machine_N$, equal to 1 if the machine is present, zero otherwise. The fixed cost per shift for Machine N is equal to \$100.

1. Clearly define all your variables and parameters consistent with the Wei and Goetschalckx model and list the legend.
2. Write down explicitly the flow conservation equations for the last warehouse before the products reach the customers (i.e., without summation signs and with numerical values for all parameters).
3. Write down explicitly the capacity constraint for the total throughput of products C and D through this final warehouse (i.e., without summation signs and with numerical values for all parameters).
4. Write the linkage or consistency constraints for assembly of product D and Machine N
5. Write the cost function for the handling and assembly of products C and D.

Recycling Operations

Consider the integrated model for the design of strategic logistics systems. SMILE is an example of such a model. The following questions are related to extending this strategic logistics model to incorporate for various additional constraints, commonly encountered in practice. In your answers, clearly indicate the bounds of any summations signs you might have used and the number of constraints of that particular type.

Using the notation developed in class for the SMILE model, write down the most compact constraint which assures that at most one transportation channel between an origin facility *i* and destination facility *j* is used, assuming there are many alternative or parallel transportation channels available between the origin and destination facility. These alternatives are indexed by *m*. Give a clear definition of the variables and parameters used in this constraint. Will there be many or few constraints of this type in the extended model?

Assume that if a warehouse is used, then it must have a minimum throughput or material flow through this warehouse. This minimum throughput is denoted by $MinFlow_{jl}$ for a warehouse of type *l* at site *j*. Write down the most compact constraint which assures that if the warehouse of that type at that site is used it will have at least the required minimum throughput. Give a clear definition of the variables and parameters used in this constraint. Will there be many or few constraints of this type in the extended model?

Consider the case where a raw material vendor *i* ships raw material *p* to a manufacturing plant *j*. The production process at the manufacturing plant creates a finished product *q* and scrap material *t*. The finished product is shipped to distribution center *k*. The scrap material is returned to the vendor from which it came to be recycled at the raw material vendor. All shipments occur at a weekly frequency. The amount of all material flows is expressed in tons per week. The production process at the plant has a 20 % scrap rate, i.e., 20 percent of the incoming raw material is scrapped. All material flows are carried by identical, company owned trucks which carry exclusively company materials. The materials are heavy so all trucks are constrained by their weight capacity only. The company can use or deploy at most *V* trucks. Write down all the required constraints to ensure that the SMILE model will find a feasible solution. Use the notation developed in class and extend in a logical fashion when required. Write the most compact constraints. Give a clear definition of the variables and parameters used in this constraint. The formulation for this case depends strongly on the detailed assumptions that are made regarding transportation. List clearly all your transportation assumptions.

Power Buy

The Savvy Customers (SC) chain of warehouse stores has been offered the opportunity for a special deal by a national manufacturer of laundry detergent. If SC purchases the expected sales quantity for the following quarter in one order and takes delivery of the complete quantity of the detergent at the beginning of the quarter, the manufacturer is offering a 25 % discount on the purchase price. This practice is commonly known as a "power buy". For the traditional, staggered purchases and deliveries, the only practical transportation mode between the manufacturing plant and the distribution center of SC is by truck. When the complete quantity is transported and delivered as a single quantity, transportation by rail has become an additional possible transportation mode. As the junior industrial engineer employed by SC you are asked to evaluate the three alternatives and make a recommendation based on which alternative has the lowest total cost.

The various quantities of detergent are given in pallets. The quarterly demand is 3000 pallets. The purchasing price when ordering in multiple orders is $120 per pallet. The holding cost rate is 25 cents on the dollar per quarter. The cost per pallet location in the warehouse is $16 per quarter. The warehouse stores have an explicit "quantities are limited" customer service policy and the stores nor the distribution center keep any safety inventory. The traditional periodic purchases occur once a week. You can assume that there are 12 weeks and that there are 90 days in a quarter. The transportation cost and transportation times for each of the three alternatives are given in the next table.

	Powerbuy Rail	Powerbuy Truck	Multibuy Truck	Units
Unit Transportation Cost ($)	4	20	24	$/pallet
Channel Transit Time (days)	14	5	5	days
Unit Purchasing Cost			$120.00	$/pallet

Summarize your calculations in a table using the traditional cost categories. Clearly separate and sum different aggregate cost categories such as invariant, variable, and fixed costs. The costs in each category must sum to the aggregate cost of the category. The sum of all aggregate costs must be equal to the total cost. You can use rows of the table to perform intermediate calculations if this clarifies your cost calculations. In the first column give the title of the cost, intermediate variable, or cost category, in the second column give the formula you used to compute this cost, in the next three columns give the computed cost for each of the three alternatives, and in the last column give the units for the variable or cost in that row. You must specify the units to get credit for the numerical answers in that row. Execute and display all cost calculations in whole dollars.

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